9 789351 766858

April 2017 | ₹ 35.00

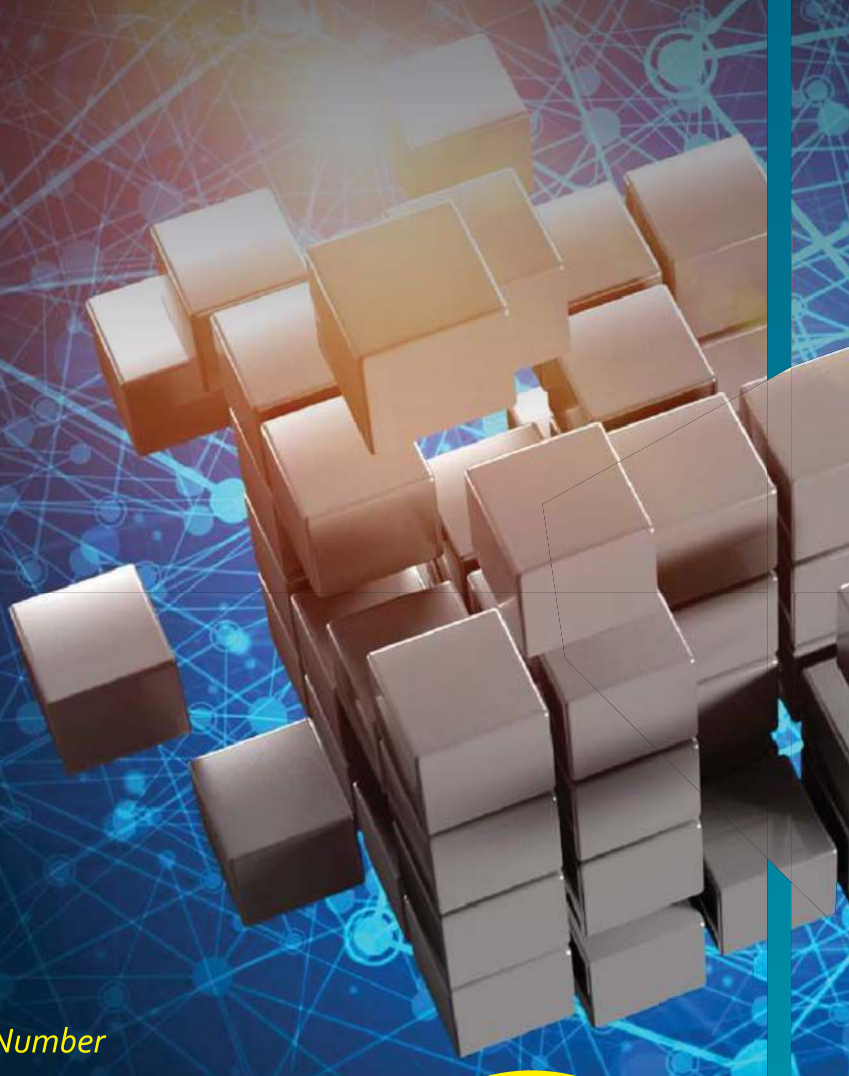
MATHEMATICS Spectrum

TARGET 2017

Test Rider for
JEE Advanced
Three Test Papers

Test Rider for
BITSAT
Three Test Papers

Test Rider for
WBJEE



JUVENILEZ

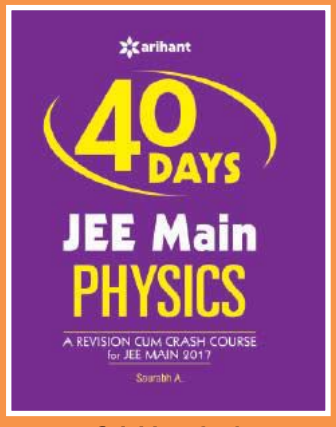
GOLDEN OLDIES *Complex Number*
TOUCH UP CLASS XII *Relation-Function*
RAINBOW *Matrix and Determinant*
FUN ARCADE

MEGA ISSUE 5
More Pages
More Features
More Practice...

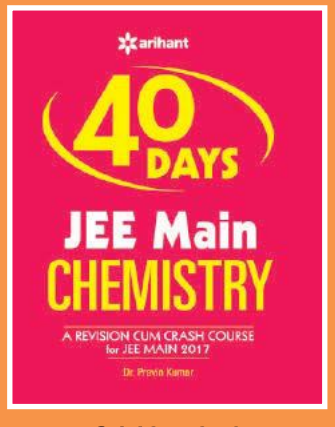
Revision *cum*
CRASH COURSES

40 DAYS
JEE MAIN 2017

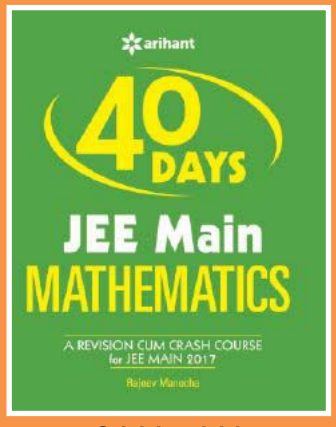
PHYSICS, CHEMISTRY, MATHEMATICS



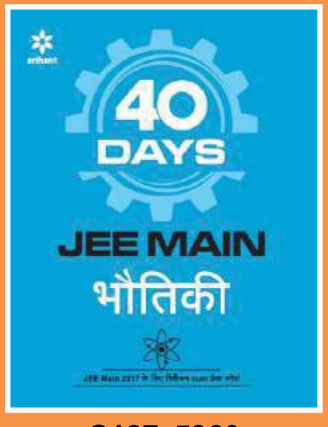
C142 ₹370



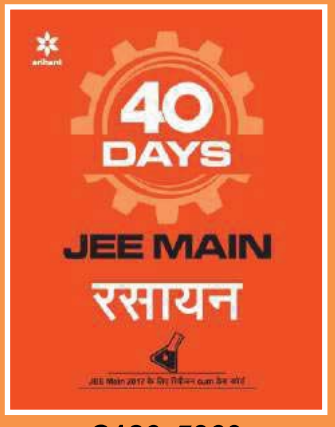
C143 ₹370



C144 ₹360



C127 ₹360



C126 ₹360

C125 ₹360

Complete Coverage of **ENGINEERING** Syllabus in
just **40 Days** with Day wise Study and **Unit Tests & Mock Tests**

MATHEMATICS Spectrum

Publisher PARUL JAIN
Chairman YC JAIN

Editorial Head SANJAY SHARMA
Publishing Manager AMIT VERMA
Project Head ZEBA SIDDIQUE
Coordinator PRACHI SINGH SAINI

Production Manager SHISHPAL SINGH

Circulation & Advertisement General Manager ANIL KUMAR GUPTA
+91-9219619948
anilgupta.arihant@gmail.com

Cover & Layout Design ATA-UR-REHMAN
Page Designer AMIT BANSAL
Diagram BRAHAMPAL SINGH
Type Setting MAYANK, SANDEEP, VIPIN, ARJUN

Copy Editor JAI PRAKASH CHAMOLA
Proof Readers GORA SHARMA, ARUN

Head Office ARIHANT MEDIA PROMOTERS
KALINDI, TP NAGAR, MEERUT-2
Phone 0121-2401479, 2512970
Fax 0121-2401648
E-mail spectrum@arihantbooks.com
Website www.arihantbooks.com

Circulation Regd. Office ARIHANT MEDIA PROMOTERS
4577/15, AGARWAL ROAD,
DARYA GANJ, NEW DELHI-2
PH. : 011-47630600

© *Publisher*
Printed and Published by Parul Jain for
Arihant Media Promoters
Printed at Arihant Publications (India) Ltd
T.P. Nagar, Meerut (U.P.)

- All rights reserved. No part of this magazine can be printed in whole or in part without the written permission of the publisher.
- The editors and publisher of this magazine do their best to verify the information published, but do not take any responsibility for the absolute accuracy of the information published.
- All disputes subject to Meerut (U.P.) jurisdiction only.

Issue 3 **Vol 5**

Dear Aspirants,

The month of April and May are the months of competitions. Three major competitions of this period are JEE Mains, BITSAT and JEE Advanced. For us, these months are most productive from creativity point of view. The month of April, especially, is the best for the effective functioning of brain with optimum weather conditions.

At the heart of calculus lies INFINITY signifying that strange region of Mathematics, which is part mystical, part magical; that's dangerous by reputation and ultimately romantic. The non-mathematical world uses 'infinity' to stand for a confusing mass of ideas. It is because of this confusion, philosophers, scientists and priests had quarreled over infinity for thousands of years and even sometimes violently.

In Mathematics on the other hand, we consider this word as a natural extension of finite. Both derivatives and integrals are the limits of infinite processes in Mathematics. We can think of them as approximation taken to its logical conclusion. Just like these two words, the world of Mathematics is full of many logical finites which silently give joy in working.

Hope all of you will work to succeed in your efforts. But remember! Success and failure entirely depends upon how you spend 24 hours of a day? Following lines of Carl Sandburg always inspire me regarding the same

"Time is the most valuable coin of your life. You and you alone will determine how that coin will be spent. Be careful that you do not let other people spend for you."

We are trying hard to provide more and more material for your practice within the purview of our limitations. The spectrum team will be anxiously waiting for your feedbacks and comments.

Good luck

Contents

Test Rider BITSAT Prep Up 01 (02)
Test Rider BITSAT Prep Up 02 (10)
Test Rider BITSAT Scale Up 03 (16)
Golden Oldies (19)
BITSAT Beats (26)
BITSAT Practice Gallery (33)
Brain Teasers (35)
Test Rider JEE Advanced Prep Up 01 (38)
Concept Map (48)
Test Rider JEE Advanced Prep Up 02 (50)
Rainbow (58)
Test Rider JEE Advanced Scale Up 03 (60)
JEE Advanced Drill (64)
Test Rider WBJEE Prep Up (73)
Touch Up Class XII (86)
Fun Arcade (94)
Quizzer (96)

TARGET BITSAT 2017

TEST RIDER

Comprehensive Simulator Test Series for BITSAT

BITSAT

(PAPER ONE PREP UP)

FULL TEST 1

(COMPLETE SYLLABUS)
FULLY SOLVED

Mock Questions from Complete Syllabus with Complete Solutions

INSTRUCTIONS

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. $\lim_{x\to 0}\left[\frac{1^x + 2^x + 3^x + \ldots + n^x}{n}\right]^{\frac{a}{x}}$ is equal to

(a) $(n!)^{a/n}$ (b) $(n!)^a$

(c) $\frac{1}{2a(n!)}$ (d) $n!$

2. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is

(a) equal to 1 (b) less than 1

(c) greater than 3 (d) equal to 3

3. If $\alpha, \beta \neq 0$, $f(n) = \alpha^n + \beta^n$ and

$$\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix}$$

$= k(1-\alpha^2)(1-\beta)^2(\alpha-\beta)^2$, then k is equal to

(a) $\alpha\beta$ (b) $\frac{1}{\alpha\beta}$ (c) 1 (d) -1

4. If S denotes the sum to infinity and S_n denotes the sum of n terms of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$, such that $S - S_n < \frac{1}{1000}$, then the least value of n is

(a) 8 (b) 9 (c) 10 (d) 11

5. The period of the function $f(x) = |\sin x| - |\cos x|$, is

(a) $\frac{\pi}{2}$ (b) π (c) 2π (d) 3π

6. If $f(x) = \begin{cases} e^x, & x \leq 0 \\ |1-x|, & x > 0 \end{cases}$, then

(a) $f(x)$ is differentiable at $x = 0$
(b) $f(x)$ is continuous at $x = 0, 1$
(c) $f(x)$ is differentiable at $x = 1$
(d) None of the above

7. The area (in sq units) in the first quadrant between $x^2 + y^2 = \pi^2$ and $y = \sin x$, is

(a) $\frac{\pi^3 - 8}{4}$ (b) $\frac{\pi^3}{4}$

(c) $\frac{\pi^3 - 16}{4}$ (d) $\frac{\pi^3 - 8}{2}$

8. A draws two cards at random from a pack of 52 cards. After returning them to pack and shuffling it, B draws two cards at random. The probability that their draws contain exactly one common card, is

(a) $\frac{25}{546}$ (b) $\frac{50}{663}$ (c) $\frac{25}{663}$ (d) $\frac{24}{563}$

9. For three events A, B and C, if P (exactly one of the events A or B occurs) $= P$ (exactly one of the events B or C occurs) $= P$ (exactly one of the events C or A occurs) $= \rho$ and P (all the three events occur simultaneously) $= \rho^2$, where $0 < \rho < 1/2$. Then, the probability of atleast one of the three events A, B and C occurring, is

(a) $\frac{3\rho + 2\rho^2}{2}$ (b) $\frac{\rho + 3\rho^2}{4}$

(c) $\frac{\rho + 3\rho^2}{2}$ (d) $\frac{3\rho + 2\rho^2}{4}$

https://vk.com/readinglecture

10. The function $f:(-\infty,-1] \to (0,e^5]$ defined by $f(x)=e^{x^3-3x+2}$, is

(a) many-one and onto
(b) many-one and into
(c) one-one and onto
(d) one-one and into

11. The constant term in the expansion $\left(1+x+\frac{2}{x}\right)^6$ is

(a) 479 (b) 517
(c) 569 (d) 581

12. The number of distinct solutions of $\sec x+\tan x=\sqrt{3}$, where $0\le x\le 3\pi$, is

(a) 1 (b) 2
(c) 3 (d) 4

13. If e_1 is the eccentricity of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$, e_2 is the eccentricity of the hyperbola passing through the foci of the ellipse and $e_1e_2=1$, then the equation of hyperbola is

(a) $\frac{x^2}{9}-\frac{y^2}{16}=1$ (b) $\frac{x^2}{16}-\frac{y^2}{9}=-1$
(c) $\frac{x^2}{9}-\frac{y^2}{25}=1$ (d) None of these

14. If A and B are two events such that $P(A)=0.6$, $P(B)=0.2$ and $P\left(\frac{A}{B}\right)=0.5$, then $P\left(\frac{A'}{B'}\right)$ is equal to

(a) $\frac{1}{10}$ (b) $\frac{3}{10}$
(c) $\frac{3}{8}$ (d) $\frac{6}{7}$

15. The sum of the infinite series $1+\frac{2}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2}+\frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3}+\ldots$, is

(a) $2^{1/3}$ (b) $4^{1/3}$ (c) $8^{1/3}$ (d) $2^{1/5}$

16. The coefficient of x^k in the expansion $\left(\frac{1-2x-x^2}{e^{-x}}\right)$ is

(a) $\frac{1-k-k^2}{k!}$ (b) $\frac{k^2+1}{k!}$
(c) $\frac{1-k}{k!}$ (d) $\frac{1}{k!}$

17. The minimum value of $Z=2x_1+3x_2$, subject to the constraints $2x_1+7x_2\ge 22$, $x_1+x_2\ge 6$, $5x_1+x_2\ge 10$ and $x_1,x_2\ge 0$, is

(a) 14 (b) 20
(c) 10 (d) 16

18. Two finite sets have m and n elements. If the total number of subsets of first set is 56 more than the total number of subsets of the second set, then the values of m and n are

(a) 7, 6 (b) 6, 3
(c) 5, 1 (d) 8, 7

19. If the harmonic mean of two numbers is 4 and their arithmetic mean A and geometric mean G satisfy the relation $2A+G^2=27$, then the numbers are

(a) 2, 6 (b) 3, 6
(c) 1, 3 (d) 1, 2

20. The angle between the lines whose direction cosines are given by $l+m+n=0$ and $l^2+m^2-n^2=0$, is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

21. The area bounded by the curve $y=xe^{|x|}$ and the lines $|x|=1$, $y=0$, is

(a) 1 sq unit (b) 2 sq units
(c) 3 sq units (d) 4 sq units

22. Let $\mathbf{a}=2\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$ and $\mathbf{b}=\hat{\mathbf{i}}+\hat{\mathbf{j}}$. If $\mathbf{c}$ is a vector such that $\mathbf{a}\cdot\mathbf{c}=|\mathbf{c}|$, $|\mathbf{c}-\mathbf{a}|=2\sqrt{2}$ and the angle between $\mathbf{a}\times\mathbf{b}$ and $\mathbf{c}$ is 30°, then $|(\mathbf{a}\times\mathbf{b})\times\mathbf{c}|$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{3\sqrt{2}}{2}$
(c) 3 (d) $\frac{3}{2}$

23. The area of the triangle (in sq units) in the argand diagram formed by the complex numbers z, iz and $z+iz$, where $z=x+iy$, is

(a) $|z|$ (b) $|z|^2$
(c) $2|z|^2$ (d) $\frac{1}{2}|z|^2$

24. A group of 6 members is chosen from 10 men and 7 women so as to contain atleast 3 men and 2 women. The number of ways this can be done, if two particular women refuse to serve on the same group, is

(a) 8000 (b) 7800
(c) 7600 (d) 7200

25. If $x+\frac{1}{x}=2$, then the principal value of $\sin^{-1}x$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{2}$ (c) π (d) $\frac{3\pi}{2}$

26. The degree of the differential equation $y(x)=1+\frac{dy}{dx}+\frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2+\frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3+\ldots$, is

(a) 2 (b) 3
(c) 1 (d) None of these

27. If the standard deviation of $y_1, y_2,\ldots, y_n$ is 3.5, then the standard deviation of $-2y_1-3, -2y_2-3,\ldots,-2y_n-3$ is

(a) −7 (b) 9
(c) 7 (d) 2.45

28. If the mean deviation about the median of the numbers $a, 2a, \ldots, 50a$ is 50, then $|a|$ is equal to

(a) 3 (b) 4
(c) 5 (d) 2

29. If the curves $\frac{x^2}{\alpha}+\frac{y^2}{4}=1$ and $y^3=16x$ intersect at right angles, then the value of α is

(a) 2 (b) $\frac{4}{3}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

30. If two lines, whose equations are $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{\lambda}$ and $\frac{x-2}{3}=\frac{y-3}{2}=\frac{z-2}{3}$ lie in the same plane. Then, the value of $\sin^{-1}(\sin\lambda)$ is

(a) 3 (b) $\pi-3$

(c) 4 (d) $\pi-4$

31. If a plane meets the coordinate axes in A, B, C and (α,β,γ) is the centroid of ΔABC. Then, the equation of the plane is

(a) $\frac{x}{3\alpha}+\frac{y}{3\beta}+\frac{z}{3\gamma}=1$ (b) $\frac{3x}{\alpha}+\frac{3y}{\beta}+\frac{3z}{\gamma}=1$

(c) $\alpha x+\beta y+\gamma z=1$ (d) None of these

32. If $a+b+c=0$, then the equation $3ax^2+2bx+c=0$ has

(a) atleast one real root in $(0, 1)$

(b) one root in $(-1, 0)$ and other is $(1, 2)$

(c) both imaginary roots

(d) two coincident roots

33. If $F(x)=f(x)\cdot g(x)$ and $f'(x)\cdot g'(x)=c$, then

(a) $F'(x)=c\left(\frac{f(x)}{f'(x)}+\frac{g(x)}{g'(x)}\right)$

(b) $F'(x)=c\left(\frac{f(x)}{f'(x)}-\frac{g(x)}{g'(x)}\right)$

(c) $\frac{F''(x)}{F(x)}=\frac{f''(x)}{f(x)}+\frac{g'(x)}{g(x)}+\frac{2c}{f(x)g(x)}$

(d) None of the above

34. If $f(x)=xe^{x(1-x)}$, then $f(x)$ is

(a) increasing in $\left[-\frac{1}{2},1\right]$

(b) decreasing in R

(c) increasing in R

(d) decreasing in $\left[-\frac{1}{2},1\right]$

35. The greatest value of $f(x)=(x+1)^{1/3}-(x-1)^{1/3}$ on $[0, 1]$ is

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{3}$

36. Let $A=[-1,1]$ and $f: A\to A$ be defined as $f(x)=x|x|$ for all $x\in A$, then the function $f(x)$ is

(a) many-one and into

(b) one-one and into

(c) many-one and onto

(d) one-one and onto

37. The x-coordinate of the incentre of the triangle that has the coordinates of mid-point of its sides as $(0, 1)$, $(1, 1)$ and $(1, 0)$, is

(a) $2+\sqrt{2}$ (b) $2-\sqrt{2}$

(c) $1+\sqrt{2}$ (d) $1-\sqrt{2}$

38. AB is chord of the circle $x^2+y^2=25$ and the tangents at A and B intersect at C. If $(2, 3)$ is the mid-point of AB, then the area of the quadrilateral $OACB$ is

(a) $50\sqrt{\frac{13}{3}}$ sq units (b) $50\sqrt{\frac{3}{13}}$ sq units

(c) $50\sqrt{3}$ sq units (d) $\frac{50}{\sqrt{3}}$ sq units

39. The number of real solutions of the equation $(5+2\sqrt{6})^{x^2-3}+(5-2\sqrt{6})^{x^2-3}=10$ is

(a) 2 (b) 4

(c) 6 (d) 8

40. Let $f(x)=\int_1^x\sqrt{2-t^2}\,dt$, then the real roots of the equation $x^2-f'(x)=0$ are

(a) ± 1 (b) $\pm\frac{1}{\sqrt{2}}$

(c) $\pm\frac{1}{2}$ (d) 0, 1

41. $\lim_{n\to\infty}\sin[\pi\sqrt{n^2+1}]$ is equal to

(a) ∞ (b) 0

(c) Does not exist (d) None of these

42. Let $f(x)=2^{2x-1}$ and $\phi(x)=-2^x+2x\log 2$.

If $f'(x)>\phi'(x)$, then

(a) $0<x<1$ (b) $0\le x<1$

(c) $x>0$ (d) $x\ge 0$

43. If $\int\frac{e^{4x}-1}{e^{2x}}\log\left(\frac{e^{2x}+1}{e^{2x}-1}\right)dx=\frac{t^2}{2}\log t-\frac{t^2}{4}-\frac{u^2}{2}\log u+\frac{u^2}{4}+C$, then

(a) $t=u=e^x+e^{-x}$

(b) $t=e^x-e^{-x}$, $u=e^x+e^{-x}$

(c) $t=e^x+e^{-x}$, $u=e^x-e^{-x}$

(d) None of the above

44. Let $g(x)$ be the inverse of the function $f(x)$ and $f'(x)=\frac{1}{1+x^3}$, then $g'(x)$ is equal to

(a) $\frac{1}{1+[g(x)]^3}$ (b) $\frac{1}{1+[f(x)]^3}$

(c) $1+[g(x)]^3$ (d) $1+[f(x)]^3$

45. Let $f(x)=\begin{cases}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x}, & x\neq 0\\ k, & x=0\end{cases}$

For what value of k, $f(x)$ is continuous at $x=0$?

(a) 1 (b) 0

(c) e (d) e^2

Answers with Explanation

1. ***(a)*** We have, $\lim_{x\to 0}\left[\frac{1^x + 2^x + 3^x + \ldots + n^x}{n}\right]^{a/x}$

$= e^{\lim_{x\to 0}\left[\frac{1^x + 2^x + 3^x + \ldots + n^x}{n} - 1\right]\frac{a}{x}}$

$= e^{\lim_{x\to 0}\left[\frac{1^x + 2^x + 3^x + \ldots + n^x - n}{n}\right]\frac{a}{x}}$

$= e^{\lim_{x\to 0}\left[\frac{1^x + 2^x + 3^x + \ldots + n^x - n}{x}\right]\frac{a}{n}}$

$= e^{\frac{a}{n}\lim_{x\to 0}\left[\frac{1^x - 1}{x} + \frac{2^x - 1}{x} + \frac{3^x - 1}{x} + \ldots + \frac{n^x - 1}{x}\right]}$

$= e^{\frac{a}{n}[\log 1 + \log 2 + \log 3 + \ldots + \log n]}$

$= e^{\frac{a}{n}\log n!} = e^{\log(n!)^{a/n}} = (n!)^{a/n}$

2. ***(a)*** We have, $|z_1| = |z_2| = |z_3| = 1$

Now, $|z_1| = 1 \Rightarrow |z_1|^2 = 1$

$z_1\bar{z}_1 = 1$

Similarly, $z_2\bar{z}_2 = z_3\bar{z}_3 = 1$

Now,

$$\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$\Rightarrow \quad |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 1$

$\Rightarrow \quad |\overline{z_1 + z_2 + z_3}| = 1$

$\Rightarrow \quad |z_1 + z_2 + z_3| = 1$

3. ***(c)*** Given, $f(n) = \alpha^n + \beta^n$

$\therefore \quad f(1) = \alpha + \beta,\ f(2) = \alpha^2 + \beta^2,\ f(3) = \alpha^3 + \beta^3$

and $f(4) = \alpha^4 + \beta^4$

Let $\Delta = \begin{vmatrix} 3 & 1 + f(1) & 1 + f(2) \\ 1 + f(1) & 1 + f(2) & 1 + f(3) \\ 1 + f(2) & 1 + f(3) & 1 + f(4) \end{vmatrix}$

$= \begin{vmatrix} 3 & 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 \\ 1 + \alpha + \beta & 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 \\ 1 + \alpha^2 + \beta^2 & 1 + \alpha^3 + \beta^3 & 1 + \alpha^4 + \beta^4 \end{vmatrix}$

$= \begin{vmatrix} 1\cdot 1 + 1\cdot 1 + 1\cdot 1 & 1\cdot 1 + 1\cdot\alpha + 1\cdot\beta & 1\cdot 1 + 1\cdot\alpha^2 + 1\cdot\beta^2 \\ 1\cdot 1 + 1\cdot\alpha + 1\cdot\beta & 1\cdot 1 + \alpha\cdot\alpha + \beta\cdot\beta & 1\cdot 1 + \alpha\cdot\alpha^2 + \beta\cdot\beta^2 \\ 1\cdot 1 + 1\cdot\alpha^2 + 1\cdot\beta^2 & 1\cdot 1 + \alpha^2\cdot\alpha + \beta^2\cdot\beta & 1\cdot 1 + \alpha^2\cdot\alpha^2 + \beta^2\cdot\beta^2 \end{vmatrix}$

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}\begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = (1-\alpha)^2(1-\beta^2)(\alpha-\beta)^2$

$\therefore \quad k = 1$

4. ***(d)*** We have, $S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$= \frac{1}{1 - \frac{1}{2}} = 2 \qquad \left[\because S_\infty = \frac{a}{1-r}\right]$

Also, $S_n = \frac{1\left[1 - \left(\frac{1}{2}\right)^n\right]}{1 - \frac{1}{2}} = 2 - \left(\frac{1}{2}\right)^{n-1} \qquad \left[\because S_n = \frac{a(1-r)^n}{1-r}\right]$

Now, $S - S_n = \left(\frac{1}{2}\right)^{n-1} < \frac{1}{1000} \Rightarrow n = 9$

But, $2^9 = 512 < 1000$

However, $2^{10} = 1024 > 1000$

$\therefore \quad n - 1 = 10 \Rightarrow n = 11$

5. ***(b)*** We have, $f(x) = |\sin x| - |\cos x|$

$\therefore \quad f(x + \pi) = |\sin(x + \pi)| - |\cos(x + \pi)|$

$= |-\sin x| - |-\cos x|$

$= |\sin x| - |\cos x| = f(x)$

So, $f(x)$ is periodic with period π.

6. ***(b)*** Given, $f(x) = \begin{cases} e^x, & x \le 0 \\ 1 - x, & 0 < x < 1 \\ x - 1, & x \ge 1 \end{cases}$

Now, $Rf'(0) = \lim_{h\to 0}\frac{f(0+h) - f(0)}{h} = \lim_{h\to 0}\frac{1 - h - 1}{h} = -1$

and $Lf'(0) = \lim_{h\to 0}\frac{f(0-h) - f(0)}{-h} = \lim_{h\to 0}\frac{e^{-h} - 1}{-h} = 1$

So, it is not differentiable at $x = 0$.

Similarly, it is not differentiable at $x = 1$ but it is continuous at $x = 0$ and 1.

7. ***(a)*** Area of the circle in Ist quadrant $= \frac{\pi(\pi^2)}{4} = \frac{\pi^3}{4}$

Also, area of curve $y = \sin x$ and X-axis is 2 sq units.

$\therefore$ Required area, $A = \frac{\pi^3}{4} - 2 = \left(\frac{\pi^3 - 8}{4}\right)$ sq unit

8. ***(b)*** Probability of both drawing the common cards x, $P(X)$ = (Probability of A drawing the card x and any other card y) $\times$ (Probability of B drawing the card x and a card other than y)

$= \frac{^{51}C_1}{^{52}C_2} \times \frac{^{50}C_1}{^{52}C_2}, \forall x$, where x has 52 value.

$\therefore$ Required probability $= \Sigma P(X)$

$= 52 \times \frac{51 \times 50 \times 4}{52 \times 51 \times 52 \times 51} = \frac{50}{663}$

9. ***(a)*** We know that, P(exactly one of A or B occurs)

$= P(A) + P(B) - 2P(A \cap B)$

Therefore, $P(A) + P(B) - 2P(A \cap B) = \rho$ ...(i)

Similarly, $P(B) + P(C) - 2P(B \cap C) = \rho$ ...(ii)

and $P(C) + P(A) - 2P(C \cap A) = \rho$ ...(ii)

On adding Eqs. (i), (ii) and (iii), we get

$2[P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A)] = 3\rho$

$\Rightarrow \quad P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) = \frac{3\rho}{2}$ ...(iv)

We are also given that

$P(A \cap B \cap C) = \rho^2$ ...(v)

https://vk.com/readinglecture

Now, P(atleast one of A, B and C occurs)
$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
$= \frac{3\rho}{2} + \rho^2$ [from Eqs. (iv) and (v)]
$= \frac{3\rho + 2\rho^2}{2}$

10. ***(d)*** We have, $f(x) = e^{x^3 - 3x + 2}$
Let $h(x) = x^3 - 3x + 2$
$\therefore \quad h'(x) = 3x^2 - 3 = 3(x^2 - 1)$
$\Rightarrow \quad h'(x) \geq 0$ for $x \in (-\infty, 1]$
$\Rightarrow f(x)$ is an increasing function.
$\therefore f(x)$ is one-one function.
Now, range of $f(x) = (0, e^4]$
But codomain of $f(x) = (0, e^5]$
$\therefore f(x)$ is an into function.
Hence, $f(x)$ is one-one and into function.

11. ***(d)*** We have,
$$\left(1 + x + \frac{2}{x}\right)^6 = 1 + {}^6C_1\left(x + \frac{2}{x}\right) + {}^6C_2\left(x + \frac{2}{x}\right)^2 + \ldots + {}^6C_6\left(x + \frac{2}{x}\right)^6$$
$\therefore$ Constant term
$= 1 + {}^6C_2 \times {}^2C_1 \times 2^1 + {}^6C_4 \times {}^4C_2 \times 2^2 + {}^6C_6 \times {}^6C_3 \times 2^3$
$= 1 + 60 + 360 + 160 = 581$

12. ***(b)*** Given, $\sec x + \tan x = \sqrt{3}$
$\Rightarrow \quad 1 + \sin x = \sqrt{3}\cos x$
$\Rightarrow \quad \sqrt{3}\cos x - \sin x = 1$
On dividing both sides by 2, we get
$$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \frac{1}{2}$$
$$\Rightarrow \quad \cos\left(x + \frac{\pi}{6}\right) = \cos\frac{\pi}{3}$$
$$\Rightarrow \quad x + \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{3}$$
$$\Rightarrow \quad x = 2n\pi + \frac{\pi}{6}, 2n\pi - \frac{\pi}{2}$$
When $x = 2n\pi + \frac{\pi}{6}$, there are solutions for $n = 0$, 1 and when $x = 2n\pi - \frac{\pi}{2}$, it does not satisfy the equation.
$\therefore$ Total number of solutions is 2.

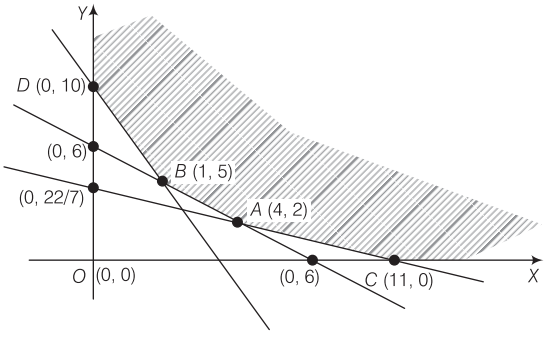
13. ***(d)*** Here, $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$
$\therefore \quad e_1 e_2 = 1 \Rightarrow e_2 = \frac{5}{3}$
Since, foci of ellipse are $(0, \pm 3)$.
Hence, equation of hyperbola is $\frac{x^2}{25} - \frac{y^2}{9} = -1$.

14. ***(c)*** $\because P(A \cap B) = P(A/B)\, P(B) = 0.5 \times 0.2 = 0.1$
$$\therefore P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)}$$
$$= \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - P(B)}$$
$$= \frac{1 - 0.6 - 0.2 + 0.1}{1 - 0.2} = \frac{3}{8}$$

15. ***(b)*** Let $S = 1 + \frac{2}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2} + \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{8}{9}\cdot\frac{1}{2^3} + \ldots$
We know that,
$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$
On comparing these two, we get
$$nx = \frac{2}{3}\cdot\frac{1}{2} \quad \ldots(i)$$
and $$\frac{n(n-1)}{2\cdot 1}x^2 = \frac{2}{3}\cdot\frac{5}{6}\cdot\frac{1}{2^2} \quad \ldots(ii)$$
From Eqs. (i) and (ii),
$$\frac{\frac{n(n-1)}{2\cdot 1}}{n^2} = \frac{\frac{2}{3}\times\frac{5}{6}\times\frac{1}{4}}{\frac{2}{3}\times\frac{1}{2}\times\frac{2}{3}\times\frac{1}{2}}$$
$$\Rightarrow \quad \frac{n-1}{2n} = \frac{5}{4} \Rightarrow n = -\frac{2}{3}$$
On putting the value of n in Eq. (i), we get
$$x = -\frac{1}{2}$$
$\therefore$ Sum of series $= \left(1 - \frac{1}{2}\right)^{-2/3} = 4^{1/3}$

16. ***(a)*** We have, $(1 - 2x - x^2)e^x$
$$= (1 - 2x - x^2)\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^k}{k!} + \ldots\right)$$
$$= (1 - 2x - x^2)\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^{k-2}}{(k-2)!} + \frac{x^{k-1}}{(k-1)!} + \frac{x^k}{k!} + \ldots\right)$$
$\therefore$ Coefficient of $x^k = \frac{1}{k!} - \frac{2}{(k-1)!} - \frac{1}{(k-2)!}$
$$= \frac{1}{k!} - \frac{2k}{k!} - \frac{k(k-1)}{k!} = \frac{1 - 2k - k^2 + k}{k!}$$
$$= \frac{1 - k - k^2}{k!}$$

17. ***(a)*** Given, $Z = 2x_1 + 3x_2$



At $D(0, 10)$, $Z = 2(0) + 3(10) = 30$
At $B(1, 5)$, $Z = 2(1) + 3(5) = 17$
At $A(4, 2)$, $Z = 2(4) + 3(2) = 14$
At $C(11, 0)$, $Z = 2(11) + 3(0) = 22$
Hence, the minimum value of Z is 14.

18. ***(b)*** Since, $2^m - 2^n = 56 = 8 \times 7 = 2^3 \times 7$

$\Rightarrow \quad 2^n(2^{m-n} - 1) = 2^3 \times 7$

$\Rightarrow \quad n = 3$ and $2^{m-n} = 8$

$\Rightarrow \quad m - n = 3$

$\Rightarrow \quad m = 6$

$\therefore \quad m = 6, n = 3$

19. ***(b)*** Let the numbers be a and b.

If H is the harmonic mean, then

$H = 4$ and $2A + G^2 = 27$ [given]

...(i)

But we know that, $AH = G^2$

$\therefore \quad 4A = G^2$

From Eq. (i), $2A + 4A = 27$

$\Rightarrow \quad A = \frac{27}{6} = \frac{9}{2}$

$\Rightarrow \quad \frac{a+b}{2} = \frac{9}{2} \quad \left[\because A = \frac{a+b}{2}\right]$

$\Rightarrow \quad a + b = 9$ ...(ii)

Again, $H = \frac{2ab}{a+b}$

$\Rightarrow \quad 4 = \frac{2ab}{9} \quad [\because H = 4]$

$\therefore \quad ab = 18$ ...(iii)

From Eqs. (ii) and (iii), we get

$a - b = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{9^2 - 4 \times 18} = \pm 3$ ...(iv)

On solving Eqs. (ii) and (iv), we get

$2a = 9 \pm 3$ and $2b = 9 \mp 3$

$\therefore \quad a = 6, 3$ and $b = 3, 6$

Hence, numbers are 3 and 6.

20. ***(c)*** We have, $l^2 + m^2 - n^2 = 0$

$\Rightarrow \quad 1 - 2n^2 = 0 \Rightarrow n = \pm \frac{1}{\sqrt{2}}$

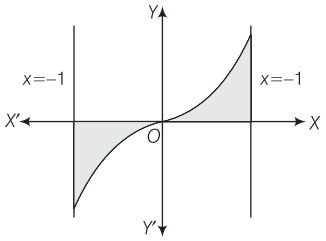
On eliminating n from the given relations, we get $lm = 0$

So, the two lines having direction cosines are $0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and $-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$.

Hence, the angle between them is

$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$.

21. ***(b)*** Required area $= 2\int_0^1 xe^x dx$



$= 2[xe^x - e^x]_0^1 = 2$ sq units

22. ***(b)*** We have, $|\mathbf{a}|^2 = 9$ and $|\mathbf{b}|^2 = 2$

$\therefore \quad |\mathbf{c} - \mathbf{a}|^2 = |\mathbf{c}|^2 + |\mathbf{a}|^2 - 2\mathbf{c}\cdot\mathbf{a} = 8$

$\Rightarrow \quad |\mathbf{c}|^2 + 9 - 2|\mathbf{c}| = 8$

$\Rightarrow \quad |\mathbf{c}| = 1$

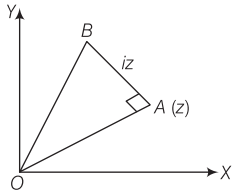
Now, $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\Rightarrow \quad |\mathbf{a} \times \mathbf{b}| = 3$

$\therefore \quad |(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}| = |\mathbf{a} \times \mathbf{b}||\mathbf{c}|\sin 45°$

$= 3 \times 1 \times \frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

23. ***(d)*** Since, $iz = ze^{i\pi/2}$. This implies that iz is the vector obtained by rotating vector z in anti-clockwise direction through 90°.



$\therefore \quad OA \perp OB$

So, area of $\Delta OAB = \frac{1}{2} \times OA \times OB = \frac{1}{2}|z||iz| = \frac{1}{2}|z|^2$

24. ***(b)*** Let the men be $M_1, M_2 \ldots, M_{10}$ and women be $W_1, W_2, \ldots W_7$.

Let W_1 and W_2 do not want to be in the same group.

The six members group can contain 4 men and 2 women or 3 men and 3 women.

The number of ways of forming $4M$, $2W$ group

$= {}^{10}C_4({}^5C_2 + 2\,{}^5C_1) = 4200$

where, ${}^5C_2$ is the number of ways without W_1 and W_2 and ${}^5C_1$ is the number of ways with W_1 and without W_2 or with W_2 and without W_1.

The number of ways of forming $3M$, $3W$ group

$= {}^{10}C_3({}^5C_3 + 2\,{}^5C_2) = 3600$

where, ${}^5C_3$ is the number of ways without W_1 and W_2 and ${}^5C_2$ is the number of ways with W_1 or W_2 but not both.

$\therefore$ Number of ways $= 4200 + 3600 = 7800$

25. ***(b)*** Given, $x + \frac{1}{x} = 2$

$\Rightarrow \quad \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2 = 0 \Rightarrow \frac{x-1}{\sqrt{x}} = 0 \Rightarrow x = 1$

So, the principal value of $\sin^{-1} x$ is $\frac{\pi}{2}$.

26. ***(c)*** Given,

$$y(x) = 1 + \frac{dy}{dx} + \frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3 + \ldots$$

$$\Rightarrow \quad y(x) = 1 + \frac{1}{1!}\cdot\frac{dy}{dx} + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + \ldots$$

$$\Rightarrow \quad y(x) = e^{dy/dx}$$

$$\Rightarrow \quad \log y(x) = \frac{dy}{dx}$$

Hence, the degree of this equation is 1.

https://vk.com/readinglecture

27. ***(c)*** Let new observations be $x_1, x_2, \ldots, x_n$ such that

$$x_i = -2y_i - 3$$

$$\therefore \quad \sigma_X = |-2|\sigma_Y = 2 \times 3.5 = 7$$

28. ***(b)*** Median of $a, 2a, 3a, 4a, \ldots, 50a$

$$= \frac{25a + 26a}{2} = 25.5a$$

Mean deviation about median

$$= \frac{\sum_{i=1}^{n} |\alpha_i - \text{Median}|}{n}$$

$$\Rightarrow \quad 50 = \frac{1}{50}\{2|a|(0.5 + 1.5 + 2.5 + \ldots + 24.5\}$$

$$\Rightarrow \quad 2500 = 2|a| \cdot \frac{25}{2}(0.5 + 24.5)$$

$$\therefore \quad |a| = 4$$

29. ***(b)*** Slope of the curve $\frac{x^2}{\alpha} + \frac{y^2}{4} = 1$ is $m_1 = -\frac{4x}{\alpha y}$

and slope of the curve $y^3 = 16x$ is $m_2 = \frac{16}{3y^2}$.

Now, apply the condition of perpendicularity of two curves,

i.e. $m_1 m_2 = -1$

$$\Rightarrow \quad \frac{-4x}{\alpha y} \cdot \frac{16}{3y^2} = -1$$

$$\Rightarrow \quad 64x = 3\alpha y^3$$

$$\Rightarrow \quad 64x = 3\alpha \cdot (16x) \qquad [\because y^3 = 16x]$$

$$\Rightarrow \quad \alpha = \frac{4}{3}$$

30. ***(d)*** Given lines are $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{\lambda}$

and $\frac{x-2}{3} = \frac{y-3}{2} = \frac{z-2}{3}$

Since, these lines are coplanar.

$$\therefore \quad \begin{vmatrix} 3-2 & 2-3 & 1-2 \\ 2 & 3 & \lambda \\ 3 & 2 & 3 \end{vmatrix} = 0$$

$$\Rightarrow \quad 1(9 - 2\lambda) + 1(6 - 3\lambda) - 1(4 - 9) = 0$$

$$\Rightarrow \quad 9 - 2\lambda + 6 - 3\lambda + 5 = 0$$

$$\Rightarrow \quad -5\lambda + 20 = 0$$

$$\Rightarrow \quad \lambda = 4$$

$$\therefore \sin^{-1}(\sin\lambda) = \sin^{-1}(\sin 4) = \sin^{-1}\{\sin(\pi - 4)\}$$

$$= \pi - 4$$

31. ***(a)*** The intercept form of the plane of equation is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$, where a, b and c are the intercepts made by the plane on the axes.

$$\therefore \quad \alpha = \frac{0 + 0 + a}{3}$$

$$\beta = \frac{0 + 0 + b}{3}$$

$$\gamma = \frac{0 + 0 + c}{3}$$

$\Rightarrow a = 3\alpha, b = 3\beta$ and $c = 3\gamma$

So, the equation of plane is $\frac{x}{3\alpha} + \frac{y}{3\beta} + \frac{z}{3\gamma} = 1$.

32. ***(a)*** Let $f(x) = ax^3 + bx^2 + cx$ in $[0, 1]$

Apply Rolle's theorem on $f(x)$ in $[0, 1]$

$\therefore f(x)$ has atleast one real root in $(0, 1)$.

33. ***(a)*** Given, $F(x) = f(x) \cdot g(x)$

On differentiating both sides w.r.t. x, we get

$$F'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$= f'(x) \cdot g'(x)\left[\frac{f(x)}{f'(x)} + \frac{g(x)}{g'(x)}\right]$$

$$\Rightarrow \quad F'(x) = c\left[\frac{f(x)}{f'(x)} + \frac{g(x)}{g'(x)}\right]$$

34. ***(a)*** Given, $f(x) = xe^{x(1-x)}$

$$\Rightarrow \quad f'(x) = e^{x(1-x)} + xe^{x(1-x)}(1 - 2x)$$

$$= e^{x(1-x)}(1 + x - 2x^2)$$

$$= -e^{x(1-x)}(x - 1)(2x + 1)$$

Hence, $f(x)$ is increasing in $\left[-\frac{1}{2}, 1\right]$.

35. ***(b)*** We have, $f(x) = (x + 1)^{1/3} - (x - 1)^{1/3}$

$$\therefore \quad f'(x) = \frac{1}{3}\left[\frac{1}{(x+1)^{2/3}} - \frac{1}{(x-1)^{2/3}}\right]$$

$$= \frac{(x-1)^{2/3} - (x+1)^{2/3}}{3(x^2 - 1)^{2/3}}$$

Clearly, $f'(x)$ does not exist at $x = \pm 1$.

$$\therefore \quad f'(x) = 0$$

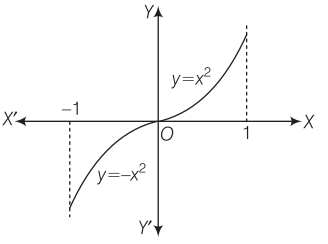
$$\Rightarrow \quad (x - 1)^{2/3} = (x + 1)^{2/3} \Rightarrow x = 0$$

Clearly, $f'(x) \neq 0$ for any other value of $x \in [0, 1]$.

So, the value of $f(x)$ at $x = 0$ is 2.

Hence, the greatest value of $f(x)$ is 2.

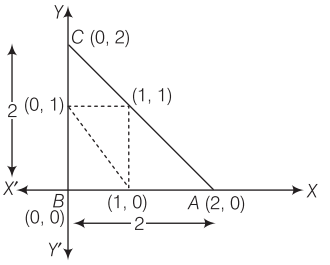
36. ***(d)*** We have, $f(x) = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



Since, $-1 \leq x \leq 1$, therefore $-1 \leq f(x) \leq 1$

$\therefore f(x)$ is one-one and onto.

37. ***(b)*** Given, mid-points of a triangle are $(0, 1)$, $(1, 1)$ and $(1, 0)$. Plot these points on a graph paper and make a triangle. So, the sides of a triangle will be 2, 2 and $\sqrt{2^2 + 2^2}$, i.e. $2\sqrt{2}$.

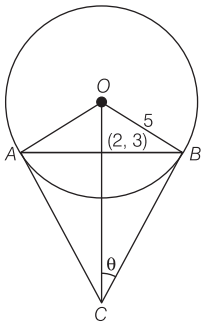


$\therefore$ x-coordinate of incentre

$$= \frac{2\cdot 0 + 2\sqrt{2}\cdot 0 + 2\cdot 2}{2 + 2 + 2\sqrt{2}} = \frac{2}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}}$$

$$= 2 - \sqrt{2}$$

38. ***(b)*** Area of quadrilateral $OACB$,



$$A = OB \cdot BC = 5^2 \cot\theta = 50\sqrt{\frac{3}{13}} \qquad \left[\because \sin\theta = \frac{\sqrt{13}}{5}\right]$$

39. ***(b)*** We have, $5 - 2\sqrt{6} = \dfrac{(5-2\sqrt{6})(5+2\sqrt{6})}{5+2\sqrt{6}} = \dfrac{1}{5+2\sqrt{6}}$

Let $(5+2\sqrt{6})^{x^2-y} = y$

Then, given equation becomes

$$y + \frac{1}{y} = 10$$

$$\Rightarrow \quad y^2 - 10y + 1 = 0$$

$$\Rightarrow \quad y = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

When $y = 2\sqrt{6} + 5$, then

$$(5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})$$

$$\Rightarrow \quad x^2 - 3 = 1$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = \pm 2$$

When $y = 5 - 2\sqrt{6}$, then

$$(5+2\sqrt{6})^{x^2-3} = (5-2\sqrt{6})$$

$$\Rightarrow \quad (5+2\sqrt{6})^{x^2-3} = (5+2\sqrt{6})^{-1}$$

$$\Rightarrow \quad x^2 - 3 = -1$$

$$\Rightarrow \quad x^2 = 2$$

$$\Rightarrow \quad x = \pm\sqrt{2}$$

$$\therefore \quad x = -2, -\sqrt{2}, \sqrt{2}, 2$$

40. ***(a)*** Given, $f(x) = \int_1^x \sqrt{2-t^2}\,dt$

$$\Rightarrow \quad f'(x) = \sqrt{2-x^2}\cdot 1 - \sqrt{2-1}\cdot 0 = \sqrt{2-x^2}$$

$$\therefore \quad x^2 = f'(x) = \sqrt{2-x^2}$$

$$\Rightarrow \quad x^4 + x^2 - 2 = 0$$

$$\Rightarrow \quad (x^2+2)(x^2-1) = 0$$

$$\Rightarrow \quad x = \pm 1$$

41. ***(b)*** We have, $\lim\limits_{n\to\infty} \sin\left[n\pi\left(1+\frac{1}{n^2}\right)^{1/2}\right]$

$$= \lim_{n\to\infty} \sin\left[n\pi\left(1 + \frac{1}{2n^2} - \frac{1}{8n^4} + \ldots\right)\right]$$

$$= \lim_{n\to\infty} \sin\left(n\pi + \frac{\pi}{2n} - \frac{\pi}{8n^3} + \ldots\right)$$

$$= \lim_{n\to\infty} (-1)^n \sin\pi\left(\frac{1}{2n} - \frac{1}{8n^3} + \ldots\right)$$

$$= 0$$

42. ***(c)*** Since, $f'(x) > \phi'(x)$

$$\Rightarrow \quad 2^{2x-1}\cdot\log 2\cdot 2 > -2^x \log 2 + 2\log 2$$

$$\Rightarrow \quad 2^{2x} > -2^x + 2$$

$$\Rightarrow \quad 2^{2x} + 2^x - 2 > 0$$

$$\Rightarrow \quad (2^x - 1)(2^x + 2) > 0$$

$$\Rightarrow \quad 2^x - 1 > 0 \qquad [\because 2^x + 2 > 0, \forall\, x]$$

$$\Rightarrow \quad 2^x > 1$$

$$\Rightarrow \quad x > 0$$

43. ***(c)*** We have, $\int \frac{e^{4x}-1}{e^{2x}} \log\left(\frac{e^{2x}+1}{e^{2x}-1}\right) dx$

$$= \int (e^{2x} - e^{-2x}) \log(e^x + e^{-x})\,dx - \int (e^{2x} - e^{-2x}) \log(e^x - e^{-x})\,dx$$

$$= \int (e^x + e^{-x}) \log(e^x + e^{-x})(e^x - e^{-x})\,dx - \int (e^x - e^{-x}) \log(e^x - e^{-x})\cdot(e^x + e^{-x})\,dx$$

$$= \int t \log t\,dt - \int u \log u\,du$$

[where, $t = e^x + e^{-x}$ and $u = e^x - e^{-x}$]

$$= \left[\log t \cdot \frac{t^2}{2} - \int \frac{1}{t}\cdot\frac{t^2}{2}\,dt\right] - \left[\log u \cdot \frac{u^2}{2} - \int \frac{1}{u}\cdot\frac{u^2}{2}\,du\right]$$

$$= \frac{t^2}{2}\log t - \frac{t^2}{4} - \frac{u^2}{2}\log u + \frac{u^2}{4} + C$$

44. ***(c)*** Since, $g(x)$ is the inverse of $f(x)$.

$$f(x) = y$$

$$\Rightarrow \quad g(y) = x$$

$$\Rightarrow \quad g'[f(x)] = \frac{1}{f'(x)}, \forall\, x$$

$$\Rightarrow \quad g'[f(x)] = 1 + x^3, \forall\, x$$

$$\Rightarrow \quad g'(y) = 1 + [g(y)]^3 \qquad [\because f(x) = y \Leftrightarrow x = g(y)]$$

$$\Rightarrow \quad g'(x) = 1 + [g(x)]^3$$

45. ***(d)*** Here, $\lim\limits_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x} = \lim\limits_{x\to 0}\left[\frac{1+\tan x}{1-\tan x}\right]^{1/x}$

$$= \lim_{x\to 0}\left[(1+\tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}} \times \lim_{x\to 0}\left[(1-\tan x)^{-\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}$$

$$= e\cdot e = e^2$$

Since, $f(x)$ is continuous at $x = 0$.

$$\therefore \quad k = e^2$$

TARGET BITSAT 2017

TEST RIDER

Comprehensive Simulator Test Series for BITSAT

BITSAT

PAPER TWO PREP UP

FULL TEST 2

(COMPLETE SYLLABUS)
FULLY SOLVED

Mock Questions from Complete Syllabus with Complete Solutions

INSTRUCTIONS

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. If $\int e^{ax} \cos bx \, dx = \frac{e^{2x}}{29} f(x) + C$, then $f(x)$ equals

(a) $2\cos 5x$ (b) $2\sin 5x$
(c) $2\cos 5x + 5\sin 5x$ (d) $2\cos 2x + 5\cos 2x$

2. For any integer n, the argument of $z = \frac{(\sqrt{3} + i)^{4n+1}}{(1 - i\sqrt{3})^{4n}}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

3. The number of distinct real values of λ, for which the vectors $-\lambda^2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - \lambda^2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \lambda^2\hat{\mathbf{k}}$ are coplanar, is

(a) 0 (b) 1 (c) 2 (d) 3

4. A teacher takes 3 children from her class to the zoo at a time as often as she can, but she does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is

(a) 12 (b) 10
(c) 60 (d) None of these

5. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$, then the value of α is

(a) ± 3 (b) ± 1 (c) ± 2 (d) ± 5

6. If the coefficients of x^7 and x^8 in $\left(2 + \frac{x}{3}\right)^n$ are equal, then n equals

(a) 56 (b) 55 (c) 45 (d) 15

7. $\cot^{-1}(2 \cdot 1^2) + \cot^{-1}(2 \cdot 2^2) + \cot^{-1}(2 \cdot 3^2) + \ldots$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{5}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

8. If a circle passes through the point (1, 2) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the equation of the locus of its centre is

(a) $x^2 + y^2 - 3x - 8y + 1 = 0$ (b) $x^2 + y^2 - 2x - 6y - 7 = 0$
(c) $2x + 4y + 9 = 0$ (d) $2x + 4y - 1 = 0$

9. The value of $\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$ is

(a) 2 (b) $\frac{3}{4}$ (c) 0 (d) None of these

10. The sum of the series $\frac{1^2}{1 \cdot 2!} + \frac{1^2 + 2^2}{2 \cdot 3!} + \frac{1^2 + 2^2 + 3^2}{3 \cdot 4!} + \ldots + \frac{1^2 + 2^2 + \ldots + n^2}{n \cdot (n+1)!} + \ldots \infty$ equals

(a) e^2 (b) $\frac{1}{2}(e + e^{-1})^2$ (c) $\frac{3e - 1}{6}$ (d) $\frac{4e + 1}{6}$

11. In how many ways can 10 true-false questions be replied?

(a) 20 (b) 100 (c) 512 (d) 1024

12. If $\log x : \log y : \log z = (y - z) : (z - x) : (x - y)$, then

(a) $x^y \cdot y^z \cdot z^x = 1$ (b) $x^x \cdot y^y \cdot z^z = 1$
(c) $\sqrt[x]{x} \cdot \sqrt[y]{y} \cdot \sqrt[z]{z} = 1$ (d) None of these

13. If a, b and c are in arithmetic progression, then the value of $(a + 2b - c)(2b + c - a)(a + 2b + c)$ is

(a) 16 abc (b) 4 abc (c) 8 abc (d) 3 abc

14. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$ and $x + y + \alpha z = \alpha - 1$ has no solution, if α is

(a) not − 2 (b) 1
(c) − 2 (d) either − 2 or 1

https://vk.com/readinglecture

15. $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1$ is equal to

(a) $\frac{n(n+1)(2n+1)}{6}$ (b) $\left(\frac{n(n+1)}{2}\right)^2$

(c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n+1)(n+2)}{6}$

16. If $k \in (-\infty, -2) \cup (2, \infty)$, then the roots of the equation $x^2 + 2kx + 4 = 0$ are

(a) complex (b) real and distinct

(c) real and equal (d) one real and one imaginary

17. If $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$, then the general value of θ is

(a) $\frac{n\pi}{3} + \frac{\pi}{12}$ (b) $\frac{n\pi}{3} + \frac{7\pi}{36}$ (c) $n\pi + \frac{7\pi}{12}$ (d) $n\pi + \frac{\pi}{12}$

18. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$, then $A \cap B$ contains

(a) one point (b) three points

(c) two points (d) four points

19. The order of the differential equation whose general solution is given by $y = c_1 e^{2x+c_2} + c_3 e^x + c_4 \sin(x + c_5)$, is

(a) 5 (b) 4 (c) 3 (d) 2

20. Three coins are tossed. If one of them shows tail, then probability that all three coins show tail, is

(a) $\frac{1}{7}$ (b) $\frac{1}{8}$ (c) $\frac{2}{7}$ (d) $\frac{1}{6}$

21. If the vectors $a\hat{i} + \hat{j} + \hat{k}$, $\hat{i} + b\hat{j} + \hat{k}$ and $\hat{i} + \hat{j} + c\hat{k}$ $(a \neq b \neq c \neq 1)$ are coplanar, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is

(a) -1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

22. The area bounded by the curve $y = |\sin x|$, X-axis and the line $|x| = \pi$ is

(a) 2 sq units (b) 3 sq units

(c) 4 sq units (d) None of these

23. If the line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ lies in the plane $4x + 4y - kz - d = 0$, then the values of k and d are

(a) 4, 8 (b) $-5, -3$

(c) 5, 3 (d) $-4, -8$

24. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The coefficient of variation is

(a) 50% (b) 10%

(c) 40% (d) None of these

25. $\tan 100° + \tan 125° + \tan 100° \tan 125°$ is equal to

(a) 0 (b) $\frac{1}{2}$ (c) -1 (d) 1

26. If the normal to the curve $y = f(x)$ at $(3, 4)$ makes an angle $\frac{3\pi}{4}$ with the positive X-axis, then $f'(3)$ is equal to

(a) -1 (b) $\frac{3}{4}$ (c) 1 (d) $-\frac{3}{4}$

27. The domain of definition of the function $y(x)$ given by $2^x + 2^y = 2$, is

(a) $(0, 1]$ (b) $[0, 1]$ (c) $(-\infty, 0]$ (d) $(-\infty, 1)$

28. If the sum of the coefficients in the expansion of $(\alpha x^2 - 2x + 1)^{35}$ is equal to the sum of the coefficients in the expansion of $(x - \alpha y)^{35}$, then α equals

(a) 0 (b) 1 (c) Any real number (d) Does not exist

29. The general solution of the differential equation $\log\left(\frac{dy}{dx}\right) = x + y$ is

(a) $e^x + e^y = C$ (b) $e^x + e^{-y} = C$

(c) $e^{-x} + e^y = C$ (d) $e^{-x} + e^{-y} = C$

30. In an ellipse, if the lines joining focus to the extremities of the minor axis form an equilateral triangle with the minor axis, then the eccentricity of the ellipse is

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{3}}{4}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{\frac{2}{3}}$

31. Rolle's theorem is applicable in the interval $-1 \leq x \leq 1$ for the function

(a) $f(x) = x$ (b) $f(x) = x^2$

(c) $f(x) = 2x^3 + 3$ (d) $f(x) = |x|$

32. If $y = \frac{a + bx^{3/2}}{x^{5/4}}$ and $y' = 0$ at $x = 5$, then the ratio $a : b$ is equal to

(a) $\sqrt{5} : 1$ (b) 5 : 2 (c) 3 : 5 (d) 1 : 2

33. If four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$ and $\frac{1}{8}$, then the probability that the problem is solved correctly by atleast one of them, is

(a) $\frac{235}{256}$ (b) $\frac{21}{256}$ (c) $\frac{3}{256}$ (d) $\frac{253}{256}$

34. The solution set of constraints $x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \leq 30$, $x \geq 0$, $y \geq 0$ includes the point

(a) (2, 3) (b) (3, 2) (c) (3, 4) (d) (4, 3)

35. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

36. If a parabola has the origin as its focus and the line $x = 2$ as the directrix, then the vertex of the parabola is at

(a) (1, 0) (b) (0, 1) (c) (2, 0) (d) (0, 2)

37. If $f(a) = 2$, $f'(a) = 1$, $g(a) = -1$ and $g'(a) = 2$, then $\lim_{x \to a} \frac{g(x) f(a) - g(a) f(x)}{x - a}$ equals

(a) 3 (b) 5 (c) 0 (d) -3

38. The point of intersection of lines $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

(a) $(-1, -1, -1)$ (b) $(-1, -1, 1)$

(c) $(1, -1, -1)$ (d) $(-1, 1, -1)$

https://vk.com/readinglecture

39. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$ attains its maximum and minimum at p and q respectively such that $p^2 = q$, then a equals
(a) 3 *(b)* 1 *(c)* 2 *(d)* 1/2

40. If $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is
(a) 1 *(b)* – 1 *(c)* 0 *(d)* 2

41. The lines $2x + y - 1 = 0$, $ax + 3y - 3 = 0$ and $3x + 2y - 2 = 0$ are concurrent for
(a) all a *(b)* $a = 4$ *(c)* $-1 \le a \le 3$ *(d)* $a > 0$

42. For any two real numbers a and b, we define aRb if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is
(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

43. If $x = 5$ and $y = -2$, then $x - 2y = 9$.
The contrapositive of this proposition is
(a) If $x - 2y = 9$, then $x \neq 5$ or $y \neq -2$
(b) If $x - 2y = 9$, then $x \neq 5$ and $y \neq -2$
(c) $x - 2y = 9$ if and only if $x = 5$ and $y = -2$
(d) None of the above

44. Let X_1 and X_2 be optimal solutions of an LPP, then
(a) $X = \lambda X_1 + (1 - \lambda)X_2$, $\lambda \in R$ is also an optimal solution
(b) $X = \lambda X_1 + (1 - \lambda)X_2$, $0 \le \lambda \le 1$, gives an optimal solution
(c) $X = \lambda X_1 + (1 + \lambda)X_2$, $0 \le \lambda \le 1$, gives an optimal solution
(d) $X = \lambda X_1 + (1 + \lambda)X_2, \lambda \in R$, gives an optional solution

45. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$, then c equals
(a) – 3 *(b)* – 1
(c) 3 *(d)* 1

Answers with Explanation

1. ***(c)*** We know that, $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2}$
$(a \cos bx + b \sin bx) + C$
$\therefore \frac{e^{2x}}{29} f(x) + C = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$
Thus, $a = 2$, $a^2 + b^2 = 29$ and $f(x) = a \cos bx + b \sin bx$
Now, $a^2 + b^2 = 29 \Rightarrow 4 + b^2 = 29 \Rightarrow b^2 = 25 \Rightarrow b = 5$
$\therefore f(x) = 2 \cos 5x + 5 \sin 5x$

2. ***(a)*** We have, $z = \frac{(\sqrt{3} + i)^{4n+1}}{(1 - i\sqrt{3})^{4n}} = \left(\frac{\sqrt{3} + i}{1 - i\sqrt{3}}\right)^{4n} (\sqrt{3} + i)$

$$= \left\{\frac{i\left(1 + \frac{\sqrt{3}}{i}\right)}{1 - i\sqrt{3}}\right\}^{4n} (\sqrt{3} + i) = \left\{\frac{i\left(1 + \frac{i\sqrt{3}}{i^2}\right)}{1 - i\sqrt{3}}\right\}^{4n} (\sqrt{3} + i)$$

$$= \left\{i\frac{(1 - i\sqrt{3})}{(1 - i\sqrt{3})}\right\}^{4n} (\sqrt{3} + i) = i^{4n}(\sqrt{3} + i)$$

$= (i^4)^n (\sqrt{3} + i) = \sqrt{3} + i$ $[\because i^4 = 1]$
$\therefore \arg(z) = \frac{\pi}{6}$

3. ***(c)*** Since, the given vectors are coplanar.

$$\therefore \begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$\Rightarrow -\lambda^2(\lambda^4 - 1) - 1(-\lambda^2 - 1) + 1(1 + \lambda^2) = 0$
$\Rightarrow -\lambda^6 + \lambda^2 + \lambda^2 + 1 + 1 + \lambda^2 = 0$
$\Rightarrow \lambda^6 - 3\lambda^2 - 2 = 0$
$\Rightarrow t^3 - 3t - 2 = 0$ [take $\lambda^2 = t$]
$\Rightarrow (t + 1)^2 (t - 2) = 0$
$\Rightarrow (\lambda^2 + 1)^2 (\lambda^2 - 2) = 0$
$\Rightarrow \lambda = \pm\sqrt{2}$

4. ***(b)*** Let there are n children in the class. Then, the number of times the teacher goes to the zoo $= {}^nC_3$
The number of times a particular child goes to the zoo $= {}^{n-1}C_2$
According to given condition, we have ${}^nC_3 = 84 + {}^{n-1}C_2$
$\Rightarrow {}^nC_3 - {}^{n-1}C_2 = 84$
$\Rightarrow {}^{n-1}C_3 = 84$ $[\because {}^{n-1}C_2 + {}^{n-1}C_3 = {}^nC_3]$
$\Rightarrow \frac{(n-1)(n-2)(n-3)}{3 \times 2 \times 1} = 84$
$\Rightarrow (n-1)(n-2)(n-3) = 6 \times 84 = 9 \times 8 \times 7$
$\Rightarrow n - 1 = 9 \Rightarrow n = 10$

5. ***(a)*** We know that, $|A^n| = |A|^n$, $n \in N$
$\therefore |A^3| = |A|^3 = 125 \Rightarrow |A| = 5$
$\Rightarrow \alpha^2 - 4 = 5 \Rightarrow \alpha^2 = 9$
$\Rightarrow \alpha = \pm 3$

6. ***(b)*** Clearly, coefficient of x^7 in $2^n\left(1 + \frac{x}{6}\right)^n$ is $2^n \cdot {}^nC_7 \cdot \left(\frac{1}{6}\right)^7$
and coefficient of x^8 is $2^n \cdot {}^nC_8 \cdot \left(\frac{1}{6}\right)^8$.
$\because$ Coefficient of x^7 = Coefficient of x^8
$\therefore 2^n \cdot {}^nC_7 \cdot \left(\frac{1}{6}\right)^7 = 2^n \cdot {}^nC_8 \cdot \left(\frac{1}{6}\right)^8 \Rightarrow \frac{{}^nC_8}{{}^nC_7} = 6$
$\Rightarrow \frac{n - 8 + 1}{8} = 6$ $\left[\because \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n - r + 1}{r}\right]$
$\Rightarrow n - 7 = 48 \Rightarrow n = 55$

7. ***(c)*** $\cot^{-1}(2x^2) = \tan^{-1}\left(\frac{1}{2x^2}\right) = \tan^{-1}\left(\frac{2}{4x^2}\right)$

$$= \tan^{-1}\left\{\frac{(2x + 1) - (2x - 1)}{1 + (2x + 1)(2x - 1)}\right\}$$

$= \tan^{-1}(2x + 1) - \tan^{-1}(2x - 1)$
Now, $\cot^{-1}(2 \cdot 1^2) + \cot^{-1}(2 \cdot 2^2) + \cot^{-1}(2 \cdot 3^2) + \ldots$
$= \tan^{-1}(3) - \tan^{-1}(1) + \tan^{-1}(5) - \tan^{-1}(3) + \ldots$
$= \tan^{-1}(\infty) - \tan^{-1}(1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

8. ***(c)*** Let the equation of circle be
$$x^2 + y^2 + 2gx + 2fy + c = 0.$$
$\because$ It passes through (1, 2), therefore $2g + 4f + 5 + c = 0$
Also, it is orthogonal w.r.t. $x^2 + y^2 = 4$.
$\therefore \quad 2g \times 0 + 2f \times 0 = c - 4$
[$\because$ condition for orthogonality is $2g_1g_2 + 2f_1f_2 = c_1 + c_2$]
$\Rightarrow \quad c = 4$
Thus, we get $28 + 4f + 9 = 0$
Hence, $2x + 4y + 9 = 0$ is the required locus.

9. ***(c)*** Let $I = \int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx$

$$= \int_0^{\pi/2} \log\left\{\frac{4 + 3\sin\left(\frac{\pi}{2} - x\right)}{4 + 3\cos\left(\frac{\pi}{2} - x\right)}\right\} dx$$

$$= \int_0^{\pi/2} \log\left(\frac{4 + 3\cos x}{4 + 3\sin x}\right) dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right)^{-1} dx = -I$$

$$\Rightarrow \quad 2I = 0 \quad \Rightarrow I = 0$$

10. ***(c)*** Let $T_n = \frac{1^2 + 2^2 + \ldots + n^2}{n(n+1)!} = \frac{n(n+1)(2n+1)}{6n(n+1)!}$

$$= \frac{2n+1}{6n!}$$

Then, $\sum_{n=1}^{\infty} T_n = \sum_{n=1}^{\infty} \frac{(2n+1)}{6n!} = \frac{1}{6}\sum_{n=1}^{\infty}\left(2\frac{1}{(n-1)!} + \frac{1}{n!}\right)$

$$= \frac{1}{6}\left(2\sum_{n=1}^{\infty}\frac{1}{(n-1)!} + \sum_{n=1}^{\infty}\frac{1}{n!}\right)$$

$$= \frac{1}{6}(2e + e - 1) = \frac{3e-1}{6}$$

11. ***(d)*** Since, every question can be answered in 2 ways, therefore required number of ways $= 2^{10} = 1024$.

12. ***(b)*** Given, $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$ (say)

$\Rightarrow \log x = k(y-z), \log y = k(z-x), \log z = k(x-y)$
$\Rightarrow x\log x + y\log y + z\log z$
$\qquad = k[x(y-z) + y(z-x) + z(x-y)]$
$\Rightarrow \log(x^x \cdot y^y \cdot z^z) = k(xy - xz + yz - xy + xz - yz)$
$\Rightarrow \log(x^x \cdot y^y \cdot z^z) = 0$
$\Rightarrow x^x \cdot y^y \cdot z^z = e^0 = 1$

13. ***(a)*** Since, a, b, c are in AP, therefore $2b = a + c$.

Now, $(a + 2b - c)(2b + c - a)(a + 2b + c)$
$= (a - c + a + c)(a + c + c - a)\cdot(a + a + c + c)$
$= (2a)(2c)\,2(a + c) = 8ac(a + c)$
$= 8ac(2b) = 16abc$

14. ***(b)*** Clearly, for no solution or infinitely many solutions,
$$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$$

$$\Rightarrow \quad \alpha(\alpha^2 - 1) - 1(\alpha - 1) + 1(1 - \alpha) = 0$$
$$\Rightarrow \quad \alpha^3 - \alpha - \alpha + 1 + 1 - \alpha = 0$$
$$\Rightarrow \quad \alpha^3 - 3\alpha + 2 = 0$$
$$\Rightarrow \quad (\alpha - 1)^2(\alpha + 2) = 0$$
$$\Rightarrow \quad \alpha = 1 \text{ or } -2$$

But for $\alpha = 1$, there are infinitely many solutions, as the given system of equation reduces to $x + y + z = 0$ which have infinite solutions.
So, the required value of α is 1.

15. ***(d)*** $\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{j} 1 = \sum_{i=1}^{n}\sum_{j=1}^{i} j = \sum_{i=1}^{n}\frac{i(i+1)}{2} = \frac{1}{2}\sum_{i=1}^{n}(i^2 + i)$

$$= \frac{1}{2}\left[\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i\right]$$
$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$
$$= \frac{1}{4}n(n+1)\left(\frac{2n+1}{3} + 1\right)$$
$$= \frac{1}{4}n(n+1)\frac{(2n+1+3)}{3}$$
$$= \frac{1}{4}n(n+1)\frac{2(n+2)}{3} = \frac{n(n+1)(n+2)}{6}$$

16. ***(b)*** Here, $D = 4k^2 - 16 = 4(k^2 - 4)$
For $k \in (-\infty, -2) \cup (2, \infty)$, $k^2 - 4 > 0$
$\therefore \quad D > 0$
Hence, the roots will be real and distinct.

17. ***(b)*** We have, $\frac{\tan 3\theta - 1}{\tan 3\theta + 1} = \sqrt{3}$

$$\Rightarrow \quad \frac{\tan 3\theta - \tan\frac{\pi}{4}}{1 + \tan 3\theta \cdot \tan\frac{\pi}{4}} = \tan\frac{\pi}{3}$$

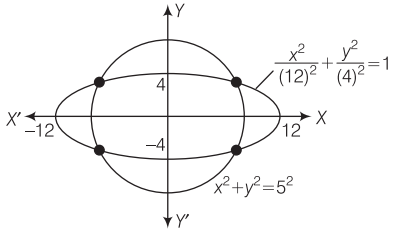
$$\Rightarrow \quad \tan\left(3\theta - \frac{\pi}{4}\right) = \tan\frac{\pi}{3} \Rightarrow 3\theta - \frac{\pi}{4} = n\pi + \frac{\pi}{3}$$

$$\Rightarrow \quad 3\theta = n\pi + \frac{\pi}{3} + \frac{\pi}{4} = n\pi + \frac{7\pi}{12}$$

$$\Rightarrow \quad \theta = \frac{n\pi}{3} + \frac{7\pi}{36}$$

18. ***(d)*** Clearly, A = set of all points lying on the circle with centre $(0, 0)$ and radius 5

B = set of all points lying on the ellipse $\frac{x^2}{(12)^2} + \frac{y^2}{(4)^2} = 1$.



From the figure, it is clear that $A \cap B$ consists of four points.

19. ***(b)*** We have, $y = c_1e^{2x + c_2} + c_3e^x + c_4\sin(x + c_5)$

$$= c_1e^{c_2}\cdot e^{2x} + c_3e^x + c_4(\sin x\cos c_5 + \cos x\sin c_5)$$
$$= Ae^{2x} + c_3e^x + B\sin x + D\cos x$$

where, $A = c_1e^{c_2}$, $B = c_4\cos c_5$ and $D = c_4\sin c_5$
Since, y contains four arbitrary constants, therefore order of differential equation = 4.

20. ***(a)*** Let E be the event that all coins shows tail and F be the event that one of them shows tail.
Then, $E = \{TTT\}$
and $\quad F = \{HHT, HTH, THH, HTT, THT, TTH, TTT\}$.

∴ Required probability

$$= P\left(\frac{E}{F}\right) = \frac{P(E \cap F)}{P(F)} = \frac{P(\{TTT\})}{P(F)} = \frac{1/8}{7/8} = \frac{1}{7}$$

21. ***(d)*** Since, the given vectors are coplanar.

$$\therefore \quad \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} a & 1 & 1 \\ 1-a & b-1 & 0 \\ 1-a & 0 & c-1 \end{vmatrix} = 0$$

On expanding, we get

$a(b-1)(c-1) - 1(1-a)(c-1) + 1[-(1-a)(b-1)] = 0$

$\Rightarrow a(b-1)(c-1) + (a-1)(c-1) + (a-1)(b-1) = 0$

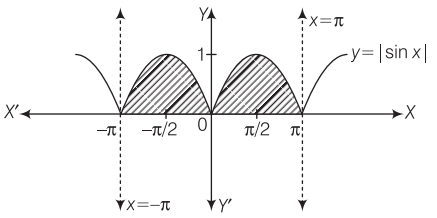
On dividing by $(a-1)(b-1)(c-1)$, we get

$$\frac{a}{a-1} + \frac{1}{b-1} + \frac{1}{c-1} = 0 \Rightarrow \frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

[multiplying both sides by -1]

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = \frac{1}{1-a} - \frac{a}{1-a} = \frac{1-a}{1-a} = 1$$

22. ***(c)*** Required area $= 2\int_0^{\pi} \sin x\, dx$



$$= 2[-\cos x]_0^{\pi} = -2[\cos \pi - \cos 0]$$

$$= -2[-1-1] = 4 \text{ sq units}$$

23. ***(c)*** Since, the line lies on the given plane, therefore any point on the line will satisfy the plane.

Clearly, any point on the line is given by $(2\lambda + 3, 3\lambda + 4, 4\lambda + 5)$.

For $\lambda = 0$, point is $(3, 4, 5)$

For $\lambda = 1$, point is $(5, 7, 9)$

Thus, the points will lie on the plane.

$\therefore \quad 12 + 16 - 5k - d = 0 \Rightarrow 5k + d = 28 \quad ...(i)$

and $\quad 20 + 28 - 9k - d = 0 \Rightarrow 9k + d = 48 \quad ...(ii)$

On solving Eqs. (i) and (ii), we get $k = 5$ and $d = 3$

24. ***(b)*** We have, $\sum_{i=1}^{10}(x_i - \bar{x})^2 = 250$ and $\bar{x} = 50$

Clearly, $\sigma = \sqrt{\dfrac{\sum_{i=1}^{10}(x_i - \bar{x})^2}{10}} = \sqrt{\dfrac{250}{10}} = 5$

$\therefore$ Coefficient of variation $= \dfrac{\sigma}{\bar{x}} \times 100 = \dfrac{5}{50} \times 100\% = 10\%$

25. ***(d)*** Clearly, $\tan(100^\circ + 125^\circ) = \dfrac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ}$

$$\Rightarrow \quad \tan(225^\circ) = \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ}$$

$$\Rightarrow \quad \tan(180^\circ + 45^\circ) = \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ}$$

$$\Rightarrow \quad 1 = \frac{\tan 100^\circ + \tan 125^\circ}{1 - \tan 100^\circ \tan 125^\circ}$$

$\Rightarrow 1 - \tan 100^\circ \tan 125^\circ = \tan 100^\circ + \tan 125^\circ$

$\Rightarrow \tan 100^\circ + \tan 125^\circ + \tan 100^\circ \tan 125^\circ = 1$

26. ***(c)*** $\because$ Slope of normal $= \dfrac{-1}{\left(\dfrac{dy}{dx}\right)_{(x_1, y_1)}}$

$$\therefore \tan\frac{3\pi}{4} = \frac{-1}{\left(\frac{dy}{dx}\right)_{(3,4)}} \Rightarrow \tan\left(\pi - \frac{\pi}{4}\right) = \frac{-1}{f'(3)}$$

$$\Rightarrow -\tan\frac{\pi}{4} = \frac{-1}{f'(3)} \Rightarrow -1 = \frac{-1}{f'(3)} \Rightarrow f'(3) = 1$$

27. ***(d)*** We have, $2^x + 2^y = 2 \Rightarrow 2^y = 2 - 2^x$

Clearly, y is real, if $2 - 2^x > 0 \Rightarrow 2 > 2^x \Rightarrow 1 > x$

$$\Rightarrow \quad x \in (-\infty, 1)$$

28. ***(b)*** According to given condition, we have

$(\alpha - 2 + 1)^{35} = (1 - \alpha)^{35} \Rightarrow (\alpha - 1)^{35} = (1 - \alpha)^{35}$

$\Rightarrow \alpha - 1 = 1 - \alpha \Rightarrow 2\alpha = 2 \quad \Rightarrow \alpha = 1$

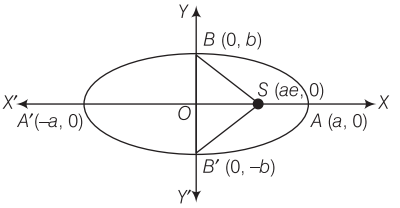
29. ***(b)*** We have, $\log\left(\dfrac{dy}{dx}\right) = x + y$

$$\Rightarrow \frac{dy}{dx} = e^{x+y} = e^x \cdot e^y \Rightarrow e^{-y}\, dy = e^x dx$$

$$\Rightarrow \frac{e^{-y}}{(-1)} = e^x + C_1 \Rightarrow e^x + e^{-y} + C_1 = 0$$

$\Rightarrow e^x + e^{-y} = C$, where $C = -C_1$

30. ***(a)*** Let the equation of ellipse be $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.



Also, let S be the focus of ellipse.

Then, $\Delta BSB'$ is an equilateral triangle and coordinates of S are $(ae, 0)$.

Clearly, area of $\Delta BSB' = \dfrac{1}{2} \times BB' \times OS = \dfrac{\sqrt{3}}{4}(\text{Side})^2$

$$\Rightarrow \frac{1}{2} \times 2b \times ae = \frac{\sqrt{3}}{4}(2b)^2 \Rightarrow abe = \frac{\sqrt{3}}{4} \times 4b^2$$

$$\Rightarrow \quad ae = \sqrt{3}\, b$$

$$\because \quad b^2 = a^2(1 - e^2)$$

$$\therefore \quad \left(\frac{ae}{\sqrt{3}}\right)^2 = a^2(1 - e^2)$$

$$\Rightarrow \quad e^2 = 3 - 3e^2 \Rightarrow e = \frac{\sqrt{3}}{2}$$

31. ***(b)*** Clearly, Rolle's theorem is not applicable for the function $f(x) = x$, as $f(1) \neq f(-1)$.

Now, consider $f(x) = x^2$

Clearly, $f(x)$ is continuous in $[-1, 1]$ and differentiable in $(-1, 1)$.

Also, $f(-1) = f(1) = 1$

$\therefore$ Rolle's theorem is applicable for $f(x) = x^2$.

32. ***(a)*** We have, $y = \dfrac{a + bx^{3/2}}{x^{5/4}}$

$$\Rightarrow \quad y' = \frac{x^{5/4}\left(b \cdot \frac{3}{2}x^{1/2}\right) - (a + bx^{3/2}) \cdot \frac{5}{4}x^{1/4}}{x^{5/2}}$$

$$\Rightarrow \quad y' = \frac{\frac{x^{1/4}}{4}\{6\,bx^{3/2} - 5(a + bx^{3/2})\}}{x^{5/2}}$$

$\because \quad y' = 0$ at $x = 5$

$\therefore\ 6\,bx^{3/2} - 5\,(a + bx^{3/2}) = 0$

$\Rightarrow \qquad 6\,bx^{3/2} - 5a - 5\,bx^{3/2} = 0$

$\Rightarrow \qquad bx^{3/2} - 5a = 0$

$\Rightarrow \qquad \dfrac{a}{b} = \dfrac{5^{3/2}}{5} = \sqrt{5} \quad \Rightarrow \quad a : b = \sqrt{5} : 1$

33. ***(a)*** P (problem solved correctly by atleast one)

$= 1 - P$ (problem is not solved by all)

$= 1 - P\,(\overline{A} \cap \overline{B} \cap \overline{C} \cap \overline{D})$

$= 1 - P(\overline{A})\,P(\overline{B})\,P(\overline{C})\,P(\overline{D})$

[if A, B, C and D are independent events, then $\overline{A}, \overline{B}, \overline{C}$ and $\overline{D}$ are also independent events]

$$= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{256 - 21}{256} = \frac{235}{256}$$

34. ***(c)*** On putting the given options in the given constraints, we get option (c) satisfy the given constraints.

35. ***(a)*** Let the roots be α and 2α.

Then, $\alpha + 2\alpha = \dfrac{1 - 3a}{a^2 - 5a + 3}$ and $\alpha \cdot 2\alpha = \dfrac{2}{a^2 - 5a + 3}$

$$\Rightarrow \alpha = \frac{1}{3} \cdot \frac{(1 - 3a)}{a^2 - 5a + 3} \text{ and } \alpha^2 = \frac{1}{a^2 - 5a + 3}$$

$$\Rightarrow \quad \frac{1}{9} \cdot \frac{(1 - 3a)^2}{(a^2 - 5a + 3)^2} = \frac{1}{a^2 - 5a + 3}$$

$\Rightarrow \quad (1 - 3a)^2 = 9(a^2 - 5a + 3)$

$\Rightarrow \quad 1 + 9a^2 - 6a = 9a^2 - 45a + 27$

$\Rightarrow \quad 39a = 26 \quad \Rightarrow a = \dfrac{2}{3}$

36. ***(a)*** Since, vertex is the mid-point of focus and intersection point of directrix and axis.

So, vertex is $\left(\dfrac{0 + 2}{2}, \dfrac{0 + 0}{2}\right) = (1, 0)$.

37. ***(b)*** We have, $\lim\limits_{x \to a} \dfrac{g(x)\,f(a) - g(a)\,f(x)}{x - a}$

$$= \lim_{x \to a} \frac{g(x)\,f(a) - f(a)\,g(a) + f(a)\,g(a) - g(a)\,f(x)}{x - a}$$

$$= \lim_{x \to a} \left\{ f(a)\left[\frac{g(x) - g(a)}{x - a}\right] - g(a)\left[\frac{[f(x) - f(a)]}{x - a}\right]\right\}$$

$= f(a)\,g'(a) - g(a)\,f'(a) = 2(2) - (-1)(1) = 4 + 1 = 5$

38. ***(a)*** Both lines are satisfied by $(-1, -1, -1)$, therefore option (a) is correct.

39. ***(c)*** We have, $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$

$\Rightarrow \quad f'(x) = 6x^2 - 18ax + 12a^2$ and $f''(x) = 12x - 18a$

For maxima and minima, put $f'(x) = 0$

$\Rightarrow \quad 6x^2 - 18ax + 12a^2 = 0 \Rightarrow x^2 - 3ax + 2a^2 = 0$

$\Rightarrow \quad (x - a)\,(x - 2a) = 0 \Rightarrow \ x = a$ or $x = 2a$

Since, $f''(a) < 0$ and $f''(2a) > 0$, therefore $x = a$ is the point of maximum and $x = 2a$ is the point of minimum.

Thus, $p = a$ and $q = 2a$

$\because \qquad p^2 = q$

$\therefore \qquad a^2 = 2a \quad \Rightarrow a = 2$ or $a = 0$

But $a > 0$, therefore $a = 2$.

40. ***(c)*** If $f(x)$ is continuous at $x = 0$, then $\lim\limits_{x \to 0} f(x) = f(0)$.

$$\Rightarrow \qquad \lim_{x \to 0} x^2 \sin\frac{1}{x} = k \qquad [\because f(0) = k]$$

$$\Rightarrow \qquad k = \lim_{x \to 0} x^2\left(\sin\frac{1}{x}\right)$$

$\Rightarrow \quad k = 0^2 \times$ (an oscillating number between -1 and 1)

$\Rightarrow \quad k = 0$

41. ***(a)*** Given lines are concurrent, if

$$\begin{vmatrix} 2 & 1 & -1 \\ a & 3 & -3 \\ 3 & 2 & -2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} 2 & 1 & 1 \\ a & 3 & 3 \\ 3 & 2 & 2 \end{vmatrix} = 0$$

[taking (-1) common from C_3]

This is true for all values of a because C_2 and C_3 are identical.

42. ***(d)*** **Reflexive** Since, $\sin^2 a + \cos^2 a = 1$ for all $a \in R$.

$\therefore \qquad aRa$

Symmetric Let aRb, then $\sin^2 a + \cos^2 b = 1$

$\Rightarrow 1 - \cos^2 a + 1 - \sin^2 b = 1 \Rightarrow \sin^2 b + \cos^2 a = 1$

$\Rightarrow \qquad bRa$

Thus, $aRb \Rightarrow bRa$

Transitive Let aRb and bRc. Then, we have

$\sin^2 a + \cos^2 b = 1$ and $\sin^2 b + \cos^2 c = 1$

On adding both the equations, we get

$\sin^2 a + (\sin^2 b + \cos^2 b) + \cos^2 c = 2$

$\Rightarrow \sin^2 a + 1 + \cos^2 c = 2 \ \Rightarrow \sin^2 a + \cos^2 c = 1$

$\Rightarrow \qquad aRc$

Hence, R is reflexive, symmetric and transitive.

$\therefore\ R$ is an equivalence relation.

43. ***(d)*** Let p, q and r be three propositions given by

$p : x = 5, q : y = -2$ and $r : x - 2y = 9$

Then, the given statement is $(p \wedge q) \to r$.

Its contrapositive is $\sim r \to \sim (p \wedge q)$

i.e. $\sim r \to \sim p \vee \sim q$

44. ***(b)*** We know that, if two points are optimal solutions, then any point on the line segment joining these two points is also an optimal solution of the LPP.

$\therefore\ X = \lambda X_1 + (1 - \lambda)X_2,\ 0 \le \lambda \le 1$, which represents the line segment joining X_1 and X_2.

45. ***(a)*** Given equation of pair of lines is

$$6x^2 - xy + 4cy^2 = 0$$

Slope of line $3x + 4y = 0$ is $-\dfrac{3}{4} = m_1$ (say)

Product of slope of lines, $m_1 m_2 = \dfrac{a}{b} = \dfrac{6}{4c} = \dfrac{3}{2c}$

$$\therefore \qquad m_2 = \frac{\frac{3}{2c}}{-\frac{3}{4}} = -\frac{2}{c}$$

Since, $\quad m_1 + m_2 = -\dfrac{2h}{b} = \dfrac{1}{4c}$

$$\therefore \qquad -\frac{3}{4} - \frac{2}{c} = \frac{1}{4c}$$

$\Rightarrow \quad -3c - 8 = 1 \quad \Rightarrow \quad c = -3$

TARGET BITSAT 2017

TEST RIDER

Comprehensive Simulator Test Series for BITSAT

BITSAT

PAPER THREE SCALE UP

Mock Questions from Complete Syllabus with Answer Key

FULL TEST 3

(COMPLETE SYLLABUS) WITH ANSWER KEY

INSTRUCTIONS

- This test consists of 45 questions and each question is allotted 3 marks for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. The integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{x}{e^x}$ (b) $\frac{e^x}{x}$ (c) xe^x (d) e^x

2. Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

(a) 1 (b) 2 (c) 3 (d) 4

3. Let $a_1, a_2, a_3, \ldots, a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i; 1 \leq p \leq 100$. For any integer n with $1 \leq n \leq 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 equals

(a) 6 (b) 5 (c) 7 (d) 4

4. If r, s, t are prime numbers and p, q are the positive integers such that LCM of p, q is $r^2 t^4 s^2$, then the number of ordered pair (p, q) is

(a) 252 (b) 254 (c) 225 (d) 224

5. The expression $[x + (x^3 - 1)^{1/2}]^5 + [x - (x^3 - 1)^{1/2}]^5$ is a polynomial of degree

(a) 5 (b) 6 (c) 7 (d) 8

6. Let a, b and c be three real numbers satisfying $[a \;\; b \;\; c]\begin{bmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{bmatrix} = [0 \;\; 0 \;\; 0]$.

If the point (a, b, c) with reference to (E) lies on the plane $2x + y + z = 1$, then the value of $7a + b + c$ is

(a) 0 (b) 12 (c) 7 (d) 6

7. The determinant $\begin{vmatrix} xp + y & x & y \\ yp + z & y & z \\ 0 & xp + y & yp + z \end{vmatrix} = 0$, if

(a) x, y, z are in AP (b) x, y, z are in GP
(c) x, y, z are in HP (d) xy, yz, zx are in AP

8. A bag contains n balls out of which some balls are white. If probability that a bag contains exactly i white ball is proportional to i^2. If a ball is drawn at random from the bag and found to be white, then find the probability that bag contains exactly 2 white balls.

(a) $\frac{\left[\frac{6}{n(n+1)}\right]\left(\frac{2^3}{n}\right)}{\frac{2}{3(n+1)}}$ (b) $\frac{\frac{2}{3(n+1)}}{\left[\frac{6}{n(n+1)}\right]\left(\frac{2^3}{n}\right)}$

(c) $\frac{\left(\frac{6}{n}\right)\left[\frac{2^3}{n(n+1)}\right]}{\frac{2}{3n}}$ (d) None of these

9. The domain of definition of $f(x) = \cos^{-1}(x + [x])$ is

(a) $[0,1)$ (b) $R - Z$
(c) $(0, \infty)$ (d) None of these

10. Let $f : R \to R$ be given by $f(x + y) = f(x) - f(y) + 2xy + 1$ for all $x, y \in R$. If $f(x)$ is everywhere differentiable and $f'(0) = 1$, then $f'(x)$ is equal to

(a) $2x + 1$ (b) $2x - 1$ (c) $x + 1$ (d) $x - 1$

11. $\frac{d^2y}{dx^2}$ is equal to

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$ (b) $-\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

12. The equation of a curve is $y = f(x)$. If the tangents at $\{1, f(1)\}, \{2, f(2)\}$ and $\{3, f(3)\}$ make angles $\frac{\pi}{6}, \frac{\pi}{3}$ and $\frac{\pi}{4}$ respectively with the positive direction of the X-axis. Then, the value of $\int_2^3 f'(x)f''(x)\,dx + \int_1^3 f''(x)\,dx$ is

(a) $-\frac{1}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$

(c) 0 (d) 1

13. If $f(x) = \int_x^{x^2} \frac{1}{(\log t)^2}\,dt;\ x \neq 0, x \neq 1$, then $f(x)$ is monotonically

(a) increasing on $(2, \infty)$ (b) increasing on $(1, 2)$

(c) decreasing on $(2, \infty)$ (d) decreasing on $(0, 3)$

14. If $\int \log(\sqrt{1-x} + \sqrt{1+x})\,dx = x f(x) + B\sin^{-1}x + C$, then

(a) $f(x) = \log(\sqrt{1-x} + \sqrt{1+x})$

(b) $A = 1/3$

(c) $B = 2/3$

(d) $B = -1/2$

15. If $I = \int_0^1 \frac{1}{1+x^{\pi/2}}\,dx$, then

(a) $\ln 2 < I < \frac{\pi}{4}$ (b) $I < \ln^2$

(c) $I > \frac{\pi}{4}$ (d) None of these

16. The area between the curve $y = 2x^4 - x^2$, the X-axis and the ordinates of two minima of the curve is

(a) 7/120 sq unit (b) 9/120 sq unit

(c) 11/120 sq unit (d) 13/120 sq unit

17. Let $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{r}$ be a variable such that $\mathbf{r}\cdot\hat{\mathbf{i}}$, $\mathbf{r}\cdot\hat{\mathbf{j}}$ and $\mathbf{r}\cdot\hat{\mathbf{k}}$ are positive integers. If $\mathbf{r}\cdot\mathbf{a} \leq 12$, then the total number of such vectors is

(a) ${}^{12}C_3$ (b) ${}^{12}C_4$ (c) ${}^{12}C_9 - 1$ (d) ${}^{12}C_8$

18. If $|\mathbf{a}| = |\mathbf{b}| = 1, |\mathbf{a} + \mathbf{b}| = \sqrt{3}$ and $\mathbf{c}$ is a vector such that $\mathbf{c} - \mathbf{a} - 2\mathbf{b} = 3(\mathbf{a} \times \mathbf{b})$, then $\mathbf{c}\cdot\mathbf{b}$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

19. If the plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ cuts the coordinate axes at A, B, C, then the area of ΔABC is

(a) $\sqrt{29}$ sq units (b) $\sqrt{41}$ sq units

(c) $\sqrt{61}$ sq units (d) $\sqrt{71}$ sq units

20. If d_1, d_2, d_3 denote the distance of the plane $2x - 3y + 4z + 2 = 0$ from the planes $2x - 3y + 4z + 6 = 0$, $4x - 6y + 8z + 3 = 0$ and $2x - 3y + 4z - 6 = 0$ respectively, then

(a) $d_1 + 8d_2 + d_3 = 0$ (b) $d_1 + 16d_2 = 0$

(c) $8d_2 = d_1$ (d) $d_1 - 2d_2 + 3d_3 = \sqrt{29}$

21. The median of the series 8, 12, 15, 7, x, 19, 22 lies in the interval

(a) [12, 15] (b) [7, 15] (c) [15, 17] (d) [9, 12]

22. The variable X takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of X, then σ^2 is equal to

(a) $\frac{f_1^2x_1^2 + f_2^2x_2^2}{f_1 + f_2} - \left(\frac{f_1x_1 + f_2x_2}{f_1 + f_2}\right)^2$

(b) $\frac{f_1f_2}{(f_1 + f_2)^2}(x_1 - x_2)^2$

(c) $\frac{f_1f_2}{(f_1 - f_2)^2}(x_1 - x_2)^2$

(d) None of the above

23. If $S = \{x \in N : 2 + \log_2\sqrt{x+1} > 1 - \log_{1/2}\sqrt{4-x^2}\}$, then

(a) $S = \{1\}$ (b) $S = Z$

(c) $S = N$ (d) None of these

24. The area of the parallelogram contained by the lines $4y - 3x - a = 0$, $3y - 4x + a = 0$, $4y - 3x - 3a = 0$ and $3y - 4x + 2a = 0$ is

(a) $\frac{2}{7}a^2$ sq unit (b) $\frac{3}{8}a^2$ sq unit

(c) $2a^2$ sq units (d) $\frac{5}{4}a^2$ sq units

25. The two circles which pass through $(0, a)$ and $(0, -a)$ touch the line $y = mx + c$, will intersect each other at right angle, if

(a) $a^2 = c^2(2m + 1)$ (b) $a^2 = c^2(2 + m^2)$

(c) $c^2 = a^2(2 + m^2)$ (d) $c^2 = a^2(2m + 1)$

26. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet the axis at P_1 and P_2. If S is the focus of the parabola, then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to

(a) $\frac{4}{a}$ (b) $\frac{2}{a}$ (c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

27. Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ and the circle $x^2 + y^2 = a^2$ at the points, where a common ordinate cuts them (on the same side of X-axis). Then, the greatest acute angle between these tangents is given by

(a) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ (b) $\tan^{-1}\left(\frac{a+b}{2\sqrt{ab}}\right)$

(c) $\tan^{-1}\left(\frac{2ab}{\sqrt{a-b}}\right)$ (d) $\tan^{-1}\left(\frac{2ab}{\sqrt{a+b}}\right)$

28. If the chords of contact of a point P with respect to a hyperbola and its auxiliary circle are at right angle. Then, the point P lies on
(a) conjugate hyperbola (b) one of the directrices
(c) one of the asymptotes (d) None of these

29. Through a point A on the X-axis, a straight line is drawn parallel to the Y-axis, so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ at B and C. If $AB = BC$, then
(a) $h^2 = 4ab$ (b) $8h^2 = 9ab$
(c) $9h^2 = 8ab$ (d) $4h^2 = ab$

30. If a and b are positive quantities such that $a > b$, then the minimum value of $a \sec\theta - b \tan\theta$ is
(a) $2ab$ (b) $\sqrt{a^2 - b^2}$ (c) $a - b$ (d) $\sqrt{a^2 + b^2}$

31. Let $x = \sin 1^\circ$, then the value of expression
$$\frac{1}{\cos 0^\circ \cdot \cos 1^\circ} + \frac{1}{\cos 1^\circ \cdot \cos 2^\circ} + \frac{1}{\cos 2^\circ \cdot \cos 3^\circ} + \ldots + \frac{1}{\cos 44^\circ \cdot \cos 45^\circ}$$ is equal to
(a) x (b) $1/x$ (c) $\sqrt{2}/x$ (d) $x/\sqrt{2}$

32. If $\tan x = n \tan y, n \in R^+$, then the maximum value of $\sec^2(x - y)$ is equal to
(a) $\frac{(n+1)^2}{2n}$ (b) $\frac{(n+1)^2}{n}$ (c) $\frac{(n+1)^2}{2}$ (d) $\frac{(n+1)^2}{4n}$

33. $\frac{\sin^3\theta - \cos^3\theta}{\sin\theta - \cos\theta} - \frac{\cos\theta}{\sqrt{1 + \cot^2\theta}} - 2\tan\theta\cot\theta = -1$, if
(a) $\theta \in \left(0, \frac{\pi}{2}\right)$ (b) $\theta \in \left(\frac{\pi}{2}, \pi\right)$
(c) $\theta \in \left(\pi, \frac{3\pi}{2}\right)$ (d) $\theta \in \left(\frac{3\pi}{2}, 2\pi\right)$

34. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then $x^4 + y^4 + z^4 + 4x^2y^2z^2 = k(x^2y^2 + y^2z^2 + z^2x^2)$, where k is equal to
(a) 1 (b) 2 (c) 3 (d) 4

35. Two medians drawn from the acute angles of a right angled triangle intersect at an angle $\pi/6$. If the length of the hypotenuse of the triangle is 3 units, then the area of triangle (in sq units) is
(a) $\sqrt{3}$ (b) 3 (c) $\sqrt{2}$ (d) 9

36. The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$, is
(a) 7 (b) 8 (c) 6 (d) 10

37. A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to reach a point P. Then, the position of P in the argand plane is
(a) $3e^{i\pi/4} + 4i$ (b) $(3 - 4i)e^{i\pi/4}$
(c) $(4 + 3i)e^{i\pi/4}$ (d) $(3 + 4i)e^{i\pi/4}$

38. If S is the set of all real x such that $\frac{2x-1}{2x^3 + 3x^2 + x}$ is positive, then S contains
(a) $(-\infty, 3/2)$ (b) $(-3/2, -1/4)$
(c) $(-1/4, 1/2)$ (d) $(1/2, 3)$

39. For any three sets A, B and C, the set $(A \cup B \cup C) \cap (A \cap B' \cap C') \cap C'$ is equal to
(a) $B \cap C'$ (b) $B' \cap C'$
(c) $B \cap C$ (d) $A \cap B \cap C$

40. Let w denotes words in the English dictionary. Define the relation R by $R = \{(x, y) \in w \times w$: the words x and y have atleast one letter in common$\}$, then R is
(a) not reflexive, symmetric and transitive
(b) not transitive, reflexive and symmetric
(c) not symmetric, reflexive and transitive
(d) reflexive, transitive and symmetric

41. Let $f : [2, \infty) \to x$ be defined by $f(x) = 4x - x^2$, then f is invertible, if x is equal to
(a) $[2, \infty)$ (b) $(-\infty, 2]$
(c) $(-\infty, 4]$ (d) $[4, \infty)$

42. Which of the following is always true?
(a) $(p \to q) \equiv (\sim q \to \sim p)$ (b) $\sim(p \vee q) \equiv (\sim p \vee \sim q)$
(c) $\sim(p \to q) \equiv (p \vee \sim q)$ (d) $\sim(p \wedge q) \equiv (\sim p \wedge \sim q)$

43. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in N$, then the least negative integral value of λ is
(a) -2 (b) -1
(c) -3 (d) -4

44. Between any two real roots of the equation $e^x \sin x - 1 = 0$, the equation $e^x \cos x + 1 = 0$ has
(a) atleast one root (b) no root
(c) atmost one root (d) exactly one root

45. If the mean and variance of a binomial variable x are 2 and 1 respectively, then the probability that x takes a value greater than or equal to 1, is
(a) $\frac{2}{3}$ (b) $\frac{4}{5}$
(c) $\frac{7}{8}$ (d) $\frac{15}{16}$

Answers

1. (b)	2. (c)	3. (a)	4. (c)	5. (c)
6. (d)	7. (b)	8. (a)	9. (a)	10. (b)
11. (b)	12. (a)	13. (a)	14. (a)	15. (a)
16. (a)	17. (a)	18. (d)	19. (c)	20. (c)
21. (a)	22. (b)	23. (a)	24. (a)	25. (c)
26. (c)	27. (a)	28. (c)	29. (b)	30. (b)
31. (b)	32. (d)	33. (b)	34. (b)	35. (a)
36. (b)	37. (d)	38. (d)	39. (a)	40. (b)
41. (c)	42. (a)	43. (b)	44. (a)	45. (d)

COMPLEX NUMBER

A collection of best questions asked in JEE to revise your concepts

1. A complex number z is said to be unimodular, if $|z| \neq 1$. If z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then, the point z_1 lies on a [2015]

(a) straight line parallel to X-axis
(b) straight line parallel to Y-axis
(c) circle of radius 2
(d) circle of radius $\sqrt{2}$

2. If z is a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$ [2014]

(a) is equal to $5/2$
(b) lies in the interval $(1, 2)$
(c) is strictly greater than $5/2$
(d) is strictly greater than $3/2$ but less than $5/2$

3. Let complex numbers α and $1/\bar{\alpha}$ lies on circles $(x - x_0)^2 + (y - y_0)^2 = r^2$ and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$, respectively. If $z_0 = x_0 + i y_0$ satisfies the equation $2|z_0|^2 = r^2 + 2$, then $|\alpha|$ is equal to [2013]

(a) $\frac{1}{\sqrt{2}}$ *(b)* $\frac{1}{2}$ *(c)* $\frac{1}{\sqrt{7}}$ *(d)* $\frac{1}{3}$

4. Let z be a complex number such that the imaginary part of z is non-zero and $a = z^2 + z + 1$ is real. Then, a cannot take the value [2012]

(a) -1 *(b)* $\frac{1}{3}$ *(c)* $\frac{1}{2}$ *(d)* $\frac{3}{4}$

5. Let $z = x + iy$ be a complex number, where x and y are integers. Then, the area of the rectangle whose vertices are the roots of the equation $z\bar{z}^3 + \bar{z}z^3 = 350$, is [2009]

(a) 48 *(b)* 32
(c) 40 *(d)* 80

6. If $|z| = 1$ and $z \neq \pm 1$, then all the values of $\frac{z}{1 - z^2}$ lie on [2007]

(a) a line not passing through the origin
(b) $|z| = \sqrt{2}$
(c) the X-axis
(d) the Y-axis

7. Let $\omega = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1$, satisfies the condition that $\left(\frac{\omega - \bar{\omega}z}{1 - z}\right)$ is purely real, then the set of values of z is [2006]

(a) $|z| = 1$ and $z \neq 2$
(b) $|z| = 1$ and $z \neq 1$
(c) $z = \bar{z}$
(d) None of the above

8. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1 + z}{1 + \bar{z}}\right)$ is equal to [2013]

(a) $-\theta$ *(b)* $\frac{\pi}{2} - \theta$
(c) θ *(d)* $\pi - \theta$

9. If $\arg(z) < 0$, then $\arg(-z) - \arg(z)$ equals [2000]

(a) π *(b)* $-\pi$
(c) $-\pi/2$ *(d)* $\pi/2$

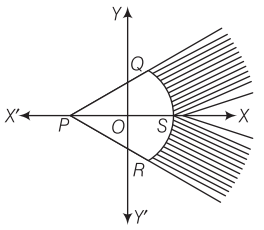
10. A particle P starts from the point $z_0 = 1 + 2i$, where $i = \sqrt{-1}$. It moves first horizontally away from origin by 5 units and then vertically away from origin by 3 units to reach a point z_1. From z_1, the particle moves $\sqrt{2}$ units in the direction of the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and then it moves through an angle $\frac{\pi}{2}$ in anti-clockwise direction on a circle with centre at origin, to reach a point z_2. The point z_2 is given by [2008]

(a) $6 + 7i$ *(b)* $-7 + 6i$
(c) $7 + 6i$ *(d)* $-6 + 7i$

11. A man walks a distance of 3 units from the origin towards the North-East (N 45° E) direction. From there, he walks a distance of 4 units towards the North-West (N 45° W) direction to reach a point P. Then, the position of P in the argand plane is [2007]

(a) $3e^{i\pi/4} + 4i$ *(b)* $(3 - 4i)\,e^{i\pi/4}$
(c) $(4 + 3i)e^{i\pi/4}$ *(d)* $(3 + 4i)\,e^{i\pi/4}$

12. The shaded region, where $P = (-1, 0)$, $Q = (-1 + \sqrt{2}, \sqrt{2})$, $R = (-1 + \sqrt{2}, -\sqrt{2})$ and $S = (1, 0)$, is represented by [2005]



(a) $|z+1|>2, |\arg(z+1)|<\frac{\pi}{4}$ (b) $|z+1|<2, |\arg(z+1)|<\frac{\pi}{2}$

(c) $|z+1|>2, |\arg(z+1)|>\frac{\pi}{4}$ (d) $|z-1|<2, |\arg(z+1)|>\frac{\pi}{2}$

13. If $0<\alpha<\frac{\pi}{2}$ is a fixed angle and $P=(\cos\theta,\sin\theta)$ and $Q=\{\cos(\alpha-\theta), \sin(\alpha-\theta)\}$, then Q is obtained from P by [2002]

(a) clockwise rotation around origin through an angle α

(b) anti-clockwise rotation around origin through an angle α

(c) reflection in the line through origin with slope $\tan\alpha$

(d) reflection in the line through origin with slope $\tan\frac{\alpha}{2}$

14. If z is a complex number lying in the first quadrant such that $\text{Re}(z)+\text{Im}(z)=3$, then the maximum value of $[\text{Re}(z)]^2\,\text{Im}(z)$ is [2001]

(a) 1 (b) 2

(c) 3 (d) 4

15. For any integer k, let $a_k=\cos\left(\frac{k\pi}{7}\right)+i\sin\left(\frac{k\pi}{7}\right)$, where $i=\sqrt{1}$. Then, the value of the expression $\frac{\sum_{k=1}^{12}|\alpha_{k+1}-\alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1}-\alpha_{4k-2}|}$ is [2015]

(a) 8 (b) 6

(c) 4 (d) 2

16. Let $\omega(\neq 1)$ be a complex cube root of unity. If $(3-3\omega+2\omega^2)^{4n+3}+(2+3\omega-3\omega^2)^{4n+3}+(-3+2\omega+2\omega^2)^{4n+3}=0$, then the set of possible value(s) of n is/are [2015]

(a) N (b) $\{3k:k\in N\}$

(c) $N-\{3k:k\in N\}$ (d) $\{6k:k\in N\}$

17. If z is a complex number of unit modulus and argument θ, then the real part of $\frac{z(1-\bar{z})}{\bar{z}(1+z)}$ is [2014]

(a) $2\cos^2\frac{\theta}{2}$ (b) $1-\cos\frac{\theta}{2}$

(c) $1+\sin\frac{\theta}{2}$ (d) $-2\sin^2\frac{\theta}{2}$

18. Let $z_k=\cos\left(\frac{2k\pi}{10}\right)+i\sin\left(\frac{2k\pi}{10}\right)$; $k=1, 2,\ldots,9$. Then, $\frac{1}{10}\{(1-z_1)(1-z_2)\ldots(1-z_9)\}$ equals [2013]

(a) 0 (b) 1

(c) 2 (d) 3

19. Let $S=S_1\cap S_2\cap S_3$, where $S_1=\{z\in C:|z|<4\}$, $S_2=\left\{z\in C:\text{Im}\left(\frac{z-1+i\sqrt{3}}{1-i\sqrt{3}}\right)\right\}$ and $S_3=\{z\in C:\text{Re}(z)>0\}$. Then, $\min|1-3i-z|$ is equal to [2013]

(a) $\frac{2-\sqrt{3}}{2}$ (b) $\frac{2+\sqrt{3}}{2}$ (c) $\frac{3-\sqrt{3}}{2}$ (d) $\frac{3+\sqrt{3}}{2}$

20. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1-z_3}{z_2-z_3}=\frac{1-i\sqrt{3}}{2}$ are the vertices of a triangle which is [2004]

(a) of area zero (b) right angled isosceles

(c) equilateral (d) obtuse angled isosceles

21. The minimum value of $|z_1-z_2|$ as z_1 and z_2 vary over the curves $|\sqrt{3}(1-2z)+2i|=2\sqrt{7}$ and $|\sqrt{3}(-1-z)-2i|=|\sqrt{3}(9-z)+18i|$ respectively, is [2012]

(a) $\frac{7\sqrt{7}}{2\sqrt{3}}$ (b) $\frac{5\sqrt{7}}{2\sqrt{3}}$ (c) $\frac{14\sqrt{7}}{\sqrt{3}}$ (d) $\frac{7\sqrt{7}}{5\sqrt{3}}$

22. If $\omega=e^{i2\pi/3}$ and a, b, c, x, y, z are non-zero complex numbers such that $a+b+c=x$, $a+b\omega+c\omega^2=y$ and $a+b\omega^2+c\omega=z$. Then, the value of $\frac{|x|^2+|y|^2+|z|^2}{|a|^2+|b|^2+|c|^2}$ is [2011]

(a) 3 (b) 6 (c) 9 (d) 1

23. The set $\left\{\text{Re}\left(\frac{2iz}{1-z^2}\right):z \text{ is a complex number}, |z|=1, z\neq\pm 1\right\}$ is [2011]

(a) $(-\infty,-1)\cup(1,\infty)$ (b) $(-\infty,0)\cup(0,\infty)$

(c) $[2,\infty)$ (d) $(-\infty,-1]\cup[1,\infty)$

24. Let ω be the solution of $x^3-1=0$ with $\text{Im}(\omega)>0$. If $a=2$ with b and c satisfying $[a\ \ b\ \ c]\begin{bmatrix}1&9&7\\2&8&7\\7&3&7\end{bmatrix}=[0\ \ 0\ \ 0]$, then the value of $\frac{3}{\omega^a}+\frac{1}{\omega^b}+\frac{3}{\omega^c}$ is [2011]

(a) -2 (b) 2 (c) 3 (d) -3

25. The set of points z in the complex plane satisfying $|z-i|z||=|z+i|z||$ is contained or equal to the set of points z satisfying [2010]

(a) $\text{Im}(z)=0$ (b) $\text{Im}(z)\leq 1$ (c) $|\text{Re}(z)|\leq 2$ (d) $|z|\leq 3$

26. Let $z=\cos\theta+i\sin\theta$. Then, the value of $\sum_{m=1}^{15}\text{Im}(z^{2m-1})$ at $\theta=2^\circ$ is [2009]

(a) $\frac{1}{\sin 2^\circ}$ (b) $\frac{1}{3\sin 2^\circ}$ (c) $\frac{1}{2\sin 2^\circ}$ (d) $\frac{1}{4\sin 2^\circ}$

27. The minimum value of $|a+b\omega+c\omega^2|$, where a, b and c are not all equal integers and $\omega(\neq 1)$ is a cube root of unity, is [2008]

(a) $\sqrt{3}$ (b) 1/2 (c) 1 (d) 0

28. If $\omega(\neq 1)$ is a cube root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals [2004]

(a) (0, 1) *(b)* (1, 1) *(c)* (1, 0) *(d)* (−1, 1)

29. Points z in the complex plane satisfying $\text{Re}(z+1)^2 = |z|^2 + 1$ lie on [2006]

(a) a circle *(b)* a parabola
(c) an ellipse *(d)* a hyperbola

30. Let A, B, C be three sets of complex number as defined below.
$A = \{z : \text{Im}(z) \geq 1\}$, $B = \{z : |z-2-i| = 3\}$,
$C = \{z : \text{Re}((1-i)z) = \sqrt{2}\}$. Then, the number of elements in the set $A \cap B \cap C$ is [2009]

(a) 0 *(b)* 1
(c) 2 *(d)* ∞

Answers with Explanation

1. ***(c)*** If z is unimodular, then $|z| = 1$.

Given, z_2 is not unimodular, i.e. $|z_2| \neq 1$ and $\dfrac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular.

$$\Rightarrow \left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1 \Rightarrow |z_1 - 2z_2|^2 = |2 - z_1\bar{z}_2|^2$$

$$\Rightarrow (z_1 - 2z_2)(\bar{z}_1 - 2\bar{z}_2) = (2 - z_1\bar{z}_2)(2 - \bar{z}_1 z_2)$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2 = 4 + |z_1|^2|z_2|^2 - 2\bar{z}_1 z_2 - 2z_1\bar{z}_2$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 = 4 + |z_1|^2|z_2|^2$$

$$\Rightarrow |z_1|^2 + 4|z_2|^2 - 4 - |z_1|^2|z_2|^2 = 0$$

$$\Rightarrow |z_1|^2(1 - |z_2|^2) - 4(1 - |z_2|^2) = 0$$

$$\Rightarrow (1 - |z_2|^2)(|z_1|^2 - 4) = 0$$

$$\Rightarrow |z_1| = 2 \text{ or } |z_2| = 1$$

But $|z_2| \neq 1$

$\therefore |z_1| = 2$

Let $z_1 = x + iy \Rightarrow x^2 + y^2 = (2)^2$

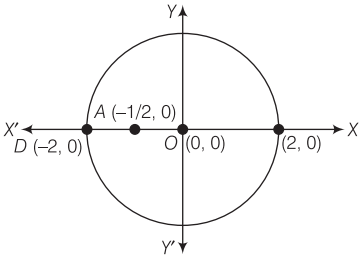
$\therefore$ Point z_1 lies on a circle of radius 2.

2. ***(b)*** Since, $|z| \geq 2$ is the region on or outside circle whose centre is (0, 0) and radius is 2.

Therefore, minimum $\left|z + \dfrac{1}{2}\right|$ is distance of z, which lie on circle $|z| = 2$ from $(-1/2, 0)$.

$\therefore$ Minimum $\left|z + \dfrac{1}{2}\right|$ = Distance of $\left(\dfrac{-1}{2}, 0\right)$ from $(-2, 0)$

$$= \sqrt{\left(-2 + \frac{1}{2}\right)^2 + 0} = \frac{3}{2}$$



Geometrically, $\min\left|z + \dfrac{1}{2}\right| = AD$

Hence, minimum value of $\left|z + \dfrac{1}{2}\right|$ lies in the interval $(1, 2)$.

3. ***(c)*** Here, $(x - x_0)^2 + (y - y_0)^2 = r^2$

and $(x - x_0)^2 + (y - y_0)^2 = 4r^2$ can be written as

$|z - z_0|^2 = r^2$ and $|z - z_0|^2 = 4r^2$

Since, α and $\dfrac{1}{\bar{\alpha}}$ lie on first and second quadrants, respectively.

$$\therefore \quad |\alpha - z_0|^2 = r^2 \text{ and } \left|\frac{1}{\bar{\alpha}} - z_0\right|^2 = 4r^2$$

$$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = r^2$$

$$\Rightarrow |\alpha|^2 - z_0\bar{\alpha} - \bar{z}_0\alpha + |z_0|^2 = r^2 \quad \ldots(i)$$

$$\text{and } \left|\frac{1}{\bar{\alpha}} - z_0\right|^2 = 4r^2 \Rightarrow \left(\frac{1}{\bar{\alpha}} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 4r^2$$

$$\Rightarrow \frac{1}{|\alpha|^2} - \frac{z_0}{\alpha} - \frac{\bar{z}_0}{\bar{\alpha}} + |z_0|^2 = 4r^2$$

Since, $|\alpha|^2 = \alpha \cdot \bar{\alpha} \Rightarrow \dfrac{1}{|\alpha|^2} - \dfrac{z_0\bar{\alpha}}{|\alpha|^2} - \dfrac{\bar{z}_0\alpha}{|\alpha|^2} + |z_0|^2 = 4r^2$

$$\Rightarrow 1 - z_0\bar{\alpha} - \bar{z}_0\alpha + |\alpha|^2|z_0|^2 = 4r^2|\alpha|^2 \quad \ldots(ii)$$

On subtracting Eq. (ii) from Eq. (i), we get

$$(|\alpha|^2 - 1) + |z_0|^2(1 - |\alpha|^2) = r^2(1 - 4|\alpha|^2)$$

$$\Rightarrow (|\alpha|^2 - 1)(1 - |z_0|^2) = r^2(1 - 4|\alpha|^2)$$

$$\Rightarrow (|\alpha|^2 - 1)\left(1 - \frac{r^2 + 2}{2}\right) = r^2(1 - 4|r|^2) \quad [\because 2|z_0|^2 = r^2 + 2]$$

$$\Rightarrow (|\alpha|^2 - 1)\left(\frac{-r^2}{2}\right) = r^2(1 - 4|\alpha|^2)$$

$$\Rightarrow |\alpha|^2 - 1 = -2 + 8|\alpha|^2 \Rightarrow 7|\alpha|^2 = 1 \Rightarrow |\alpha| = \frac{1}{\sqrt{7}}$$

4. ***(d)*** Let $z = x + iy \Rightarrow a = (x + iy)^2 + (x + iy) + 1$

$$\Rightarrow (x^2 - y^2 + x + 1 - a) + i(2xy + y) = 0 \quad \ldots(i)$$

It is purely real, if $y(2x + 1) = 0$ but imaginary part of z, i.e. y is non-zero.

$$\Rightarrow 2x + 1 = 0 \Rightarrow x = -1/2$$

From Eq. (i), $\dfrac{1}{4} - y^2 - \dfrac{1}{2} + 1 - a = 0 \Rightarrow a = -y^2 + \dfrac{3}{4} \Rightarrow a < \dfrac{3}{4}$

5. ***(a)*** Since, $z\bar{z}(z^2 + \bar{z}^2) = 350$

$$\Rightarrow 2(x^2 + y^2)(x^2 - y^2) = 350$$

$$\Rightarrow (x^2 + y^2)(x^2 - y^2) = 175$$

Since, $x, y \in I$, therefore the only possible case which gives integral solution, is

$$x^2 + y^2 = 25 \quad \ldots(ii)$$

and $x^2 - y^2 = 7 \quad \ldots(iii)$

From Eqs. (ii) and (iii),

$$x^2 = 16,\ y^2 = 9 \Rightarrow x = \pm 4,\ y = \pm 3$$

$\therefore$ Area of rectangle $= 8 \times 6 = 48$ sq units

6. ***(d)*** Let $z = \cos\theta + i\sin\theta$

$$\Rightarrow \frac{z}{1-z^2} = \frac{\cos\theta + i\sin\theta}{1-(\cos 2\theta + i\sin 2\theta)} = \frac{\cos\theta + i\sin\theta}{2\sin^2\theta - 2i\sin\theta\cos\theta}$$

$$= \frac{\cos\theta + i\sin\theta}{-2i\sin\theta(\cos\theta + i\sin\theta)} = \frac{i}{2\sin\theta}$$

Hence, $\dfrac{z}{1-z^2}$ lies on the imaginary axis, i.e. Y-axis.

7. ***(b)*** Let $z_1 = \dfrac{\omega - \bar{\omega}z}{1-z}$ be purely real.

Then, $z_1 = \bar{z}_1$

$$\therefore \frac{\omega - \bar{\omega}z}{1-z} = \frac{\bar{\omega} - \omega\bar{z}}{1-\bar{z}}$$

$$\Rightarrow \omega - \omega\bar{z} - \bar{\omega}z + \bar{\omega}z\cdot\bar{z} = \bar{\omega} - \omega\bar{z} - \bar{\omega}z + \omega z\cdot\bar{z}$$

$$\Rightarrow (\omega - \bar{\omega}) + (\bar{\omega} - \omega)|z|^2 = 0$$

$$\Rightarrow (\omega - \bar{\omega})(1 - |z|^2) = 0 \Rightarrow |z|^2 = 1 \qquad [\because \omega - \bar{\omega} \neq 0, \text{ as } \beta \neq 0]$$

$$\Rightarrow |z| = 1 \text{ and } z \neq 1$$

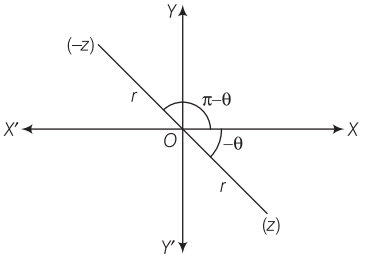
8. ***(c)*** Given, $|z| = 1$ and $\arg(z) = \theta$

$$\therefore z = e^{i\theta}$$

But $\bar{z} = \dfrac{1}{z}$

$$\therefore \arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg\left(\frac{1+z}{1+\frac{1}{z}}\right) = \arg(z) = \theta$$

9. ***(a)*** Since, $\arg(z) < 0 \Rightarrow \arg(z) = -\theta$



$$\Rightarrow z = r\cos(-\theta) + i\sin(-\theta) = r(\cos\theta - i\sin\theta)$$

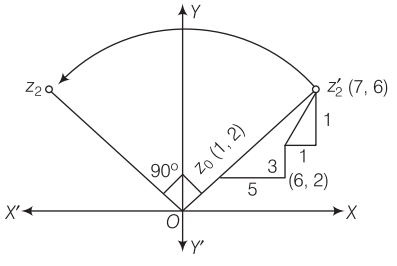
and $(-z) = -r(\cos\theta - i\sin\theta) = r[\cos(\pi - \theta) + i\sin(\pi - \theta)]$

$\therefore \arg(-z) = \pi - \theta$

Thus, $\arg(-z) - \arg(z) = \pi - \theta - (-\theta) = \pi$

10. ***(d)*** Here, $z_2' = (6 + \sqrt{2}\cos 45°, 5 + \sqrt{2}\sin 45°) = (7, 6) = 7 + 6i$

By rotation about $(0, 0)$,

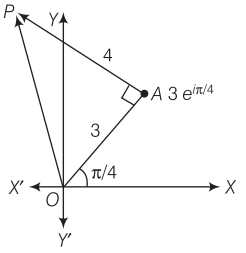


$$\frac{z_2}{z_2'} = e^{i\pi/2} \Rightarrow z_2 = z_2'\ (e^{i\pi/2})$$

$$= (7 + 6i)\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$= (7 + 6i)(i) = -6 + 7i$$

11. ***(d)*** Let $OA = 3$, so that the complex number associated with A is $3e^{i\pi/4}$.



If z is the complex number associated with P, then

$$\frac{z - 3e^{i\pi/4}}{0 - 3e^{i\pi/4}} = \frac{4}{3}e^{-i\pi/2} = -\frac{4i}{3}$$

$$\Rightarrow 3z - 9e^{i\pi/4} = 12ie^{i\pi/4}$$

$$\Rightarrow z = (3 + 4i)\,e^{i\pi/4}$$

12. ***(a)*** Since, $|PQ| = |PS| = |PR| = 2$

$\therefore$ Shaded part represents the external part of circle having centre $(-1, 0)$ and radius 2.

We know that, equation of circle having centre z_0 and radius r, is $|z - z_0| = r$

$$\therefore |z - (-1 + 0i)| > 2 \Rightarrow |z + 1| > 2$$

Also, argument of $z + 1$ with respect to positive direction of X-axis is $\pi/4$.

$$\therefore \arg(z + 1) \leq \frac{\pi}{4} \qquad \ldots(i)$$

and argument of $z + 1$ in anti-clockwise direction is $-\pi/4$.

$$\therefore -\pi/4 \leq \arg(z + 1) \qquad \ldots(ii)$$

From Eqs. (i) and (ii), $-\pi/4 \leq |\arg(z + 1)| \leq \pi/4$

13. ***(d)*** In the argand plane, P is represented by $e^{i\theta}$ and Q is represented by $e^{i(\alpha - \theta)}$.

Now, rotation about a line with angle α is given by $e^{\theta} \to e^{(\alpha - \theta)}$. Therefore, Q is obtained from P by reflection in the line making an angle $\alpha/2$.

14. ***(d)*** Let $z = a + ib$, where $a > 0, b > 0$

Then, $\text{Re}(z) + \text{Im}(z) = 3$

$$\Rightarrow a + b = 3$$

Let $P = \{\text{Re}(z)\}^2\,\text{Im}(z) = a^2 b$

Then, $P = a^2(3 - a) = 3a^2 - a^3$

$$\therefore \frac{dP}{da} = 6a - 3a^2 \text{ and } \frac{d^2P}{da^2} = 6 - 6a$$

For maximum or minimum value of P,

Put $\dfrac{dP}{da} = 0 \Rightarrow a = 0$ and $a = 2$

Clearly, $\dfrac{d^2P}{da^2} = 6 - 12 < 0$ for $a = 2$

For $a = 2$, $P = 4(3 - 2) = 4$

15. ***(c)*** We have, $\alpha_k = \cos\left(\frac{k\pi}{7}\right) + i\sin\left(\frac{k\pi}{7}\right) = e^{ik\pi/7}$

$\therefore \quad |\alpha_k| = 1$ and $\alpha_{k+1} = \alpha_1\alpha_k$, $\alpha_{4k-1} = \alpha_{4k-2}\alpha_1$

Now, $\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k| = \sum_{k=1}^{12} |\alpha_k||\alpha_1 - 1|$

$= \sum_{k=1}^{12} |\alpha_1 - 1| = 12(\alpha_1 - 1)$ and $\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|$

$= \sum_{k=1}^{3} |\alpha_{4k-2}||\alpha_1 - 1| = \sum_{k=1}^{3} |\alpha_1 - 1| = 3(\alpha_1 - 1)$

$\therefore \dfrac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|} = \dfrac{12(\alpha_1 - 1)}{3(\alpha_1 - 1)} = 4$

16. ***(c)*** We have, $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$

$\Rightarrow (3 - 3\omega + 2\omega^2)^{4n+3} + \{\omega(3 - 3\omega + 2\omega^2)\}^{4n+3} + \{\omega^2(3 - 3\omega + 2\omega^2)\}^{4n+3} = 0$

$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3}(1 + \omega^{4n+3} + \omega^{8n+6}) = 0$

$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3}(1 + \omega^{4n} + \omega^{8n}) = 0$

$\Rightarrow \quad (3 - 3\omega + 2\omega^2)^{4n+3}(1 + \omega^{n} + \omega^{2n}) = 0$

$\Rightarrow \quad 1 + \omega^n + \omega^{2n} = 0$

$\Rightarrow n$ is not a multiple of 3.

Hence, the set of values of n is $N - \{n : n = 3k, k \in N\}$.

17. ***(d)*** We have, $|z| = 1$. So, let $z = \cos\theta + i\sin\theta$

Then, $\bar{z} = \cos\theta - i\sin\theta$

$\therefore \dfrac{z(1-\bar{z})}{\bar{z}(1+z)} = \dfrac{z - |z|^2}{\bar{z} + |z|^2} = \dfrac{z-1}{\bar{z}+1} = \dfrac{(\cos\theta - 1) + i\sin\theta}{(\cos\theta + 1) - i\sin\theta}$

$= -\dfrac{2\sin^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} - 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}} = \dfrac{2i\sin\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} - i\sin\frac{\theta}{2}\right)}$

$= i\tan\dfrac{\theta}{2}(\cos\theta + i\sin\theta)$

$\therefore$ Real part of $\dfrac{z(1-\bar{z})}{\bar{z}(1+z)} = -\tan\dfrac{\theta}{2}\sin\theta = -2\sin^2\dfrac{\theta}{2}$

18. ***(b)*** Clearly, $1, z_1, z_2, \ldots, z_9$ are 10 roots of unity.

$\therefore \quad z^{10} - 1 = (z-1)(z-z_1)(z-z_2)\ldots(z-z_0)$

$\Rightarrow \quad (z-z_1)(z-z_2)\ldots(z-z_9) = \dfrac{z^{10}-1}{z-1}$

$\Rightarrow \quad (z-z_1)(z-z_2)\ldots(z-z_9) = 1 + z + z^2 + \ldots + z^9$

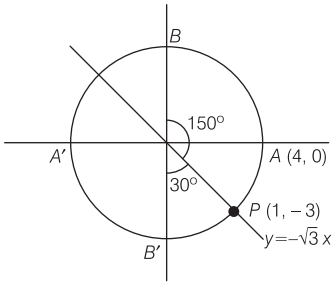
$\Rightarrow \quad (1-z_1)(1-z_2)\ldots(1-z_9) = 10$

$\Rightarrow \quad \dfrac{1}{10}\{(1-z_1)(1-z_2)\ldots(1-z_9)\} = 1$

19. ***(c)*** Clearly, $|1 - 3i - z| = |z - (1 - 3i)|$

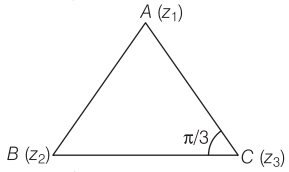
= Distance between z and $(1, -3)$

Thus, $|1 - 3i - z|$ represents distance of any point in the region represented by S from the point $P(1, -3)$. So, $|1 - 3i - z|$ is minimum when z is the foot of the perpendicular drawn from $P(1, -3)$ on the line $y = \sqrt{3}x$.



Hence, the minimum value of $|1 - 3i - z|$ is equal to the length of perpendicular drawn from $P(1, -3)$ on the line $\sqrt{3}x + y = 0$, i.e. $\left|\dfrac{\sqrt{3} - 3}{\sqrt{(\sqrt{3})^2 + 1^2}}\right| = \dfrac{3 - \sqrt{3}}{2}$.

20. ***(c)*** We have, $\dfrac{z_1 - z_3}{z_2 - z_3} = \dfrac{1 - i\sqrt{3}}{2}$



$\Rightarrow \quad (z_1 - z_3) = e^{-i\pi/3}(z_2 - z_3)$

$\Rightarrow CA$ is obtained by rotating CB through $\pi / 3$ is clockwise sense.

$\Rightarrow CA = CB$ and $\angle ABC = 60°$

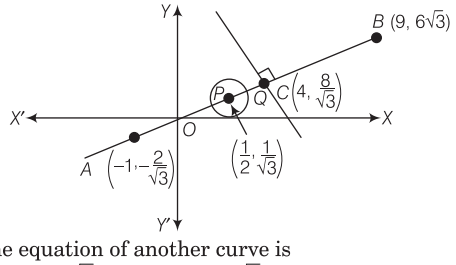
$\Rightarrow \Delta ABC$ is an equilateral triangle.

21. ***(b)*** We have,

$|\sqrt{3}(1 - 2z) + 2i| = 2\sqrt{7} \Rightarrow |-2\sqrt{3}z + \sqrt{3} + 2i| = 2\sqrt{7}$

$\Rightarrow \quad \left|z - \left(\dfrac{1}{2} + \dfrac{1}{\sqrt{3}}i\right)\right| = \dfrac{\sqrt{7}}{3} \qquad \ldots(i)$

Clearly, it represents a circle having centre at $\left(\dfrac{1}{2}, \dfrac{1}{\sqrt{3}}\right)$ and radius $r_1 = \sqrt{\dfrac{7}{3}}$. It is given that z_1 lies on Eq. (i).



The equation of another curve is

$|\sqrt{3}(-1 - z) - 2i| = |\sqrt{3}(9 - z) + 18i|$

$\Rightarrow \quad \left|-1 - z\dfrac{2}{\sqrt{3}}i\right| = |9 - z + 6\sqrt{3}i|$

$\Rightarrow \quad \left|z + 1 + \dfrac{2}{\sqrt{3}}i\right| = |z - 9 - 6\sqrt{3}i|$

$\Rightarrow \quad \left|z - \left(-1 - \dfrac{2}{\sqrt{3}}i\right)\right| = |z - (9 + 6\sqrt{3}i)|$

This represents perpendicular bisector of the line segment joining points $A\left(-1, \dfrac{-2}{\sqrt{3}}\right)$ and $B(9, 6\sqrt{3})$. The coordinates

of the mid-point C of AB are $(4, 8/\sqrt{3})$. Clearly, A, B and the centre of the circle are collinear.

$\therefore \quad \min|z_1 - z_2| = QC = CP - PQ = \frac{7}{2}\sqrt{\frac{7}{3}} - \sqrt{\frac{7}{3}} = \frac{5}{2}.\sqrt{\frac{7}{3}}$

22. ***(a)*** We have, $x = a + b + c$

$\Rightarrow \quad |x|^2 = x\bar{x} = (a+b+c)(\bar{a}+\bar{b}+\bar{c})$

$\Rightarrow \quad |x|^2 = a\bar{a} + b\bar{b} + c\bar{c} + (a\bar{b} + \bar{a}b) + (b\bar{c} + \bar{b}c) + (c\bar{a} + \bar{c}a)$

and $\quad y = a + b\omega + c\omega^2$

$\Rightarrow \quad |y|^2 = y\bar{y} = (a + b\omega + c\omega^2)\overline{(a + b\omega + c\omega^2)}$

$= (a + b\omega + c\omega^2)(\bar{a} + \bar{b}\omega^2 + \bar{c}\omega)$

$\Rightarrow \quad |y|^2 = a\bar{a} + b\bar{b} + c\bar{c} + (a\bar{b}\omega^2 + \bar{a}b\omega)$

$+ (b\bar{c}\omega^2 + \bar{b}c\omega) + (\bar{c}a\omega + c\bar{a}\omega^2)$

$\Rightarrow \quad |y|^2 = |a|^2 + |b|^2 + |c|^2 + (a\bar{b}\omega^2 + \bar{a}b\omega)$

$+ (b\bar{c}\omega^2 + \bar{b}c\omega) + (\bar{c}a\omega + c\bar{a}\omega^2)$

Similarly, $|z|^2 = |a|^2 + |b|^2 + |c|^2 + (a\bar{b}\omega + \bar{a}\,b\omega^2)$

$(b\bar{c}\omega + \bar{b}c\omega^2) + (\bar{c}a\omega^2 + c\bar{a}\omega)\,\bar{a}b$

$\therefore |x|^2 + |y|^2 + |z|^2 = 3\{|a|^2 + |b|^2 + |c|^2\}$

$\Rightarrow \quad \frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = 3$

23. ***(a)*** Let $z = x + iy$

Then, $\frac{2iz}{1-z^2} = \frac{2i(x+iy)}{1-(x+iy)^2} = \frac{-2y + 2ix}{1-(x^2 - y^2 + 2ixy)}$

$\Rightarrow \quad \frac{2iz}{1-z^2} = \frac{-2y + 2ix}{2y^2 - 2ixy}$

$[\because |z| = 1 \Rightarrow x^2 + y^2 = 1 \Rightarrow 1 - x^2 = y^2]$

$\Rightarrow \quad \frac{2iz}{1-z^2} = -\frac{1}{y}$

$\therefore \left\{\text{Re}\left(\frac{2iz}{1-z^2}\right) : z \text{ is a complex number}, |z| = 1, z \neq \pm 1\right\}$

$= \left\{-\frac{1}{y} : x^2 + y^2 = 1, x \neq \pm 1\right\} = \left\{\pm\frac{1}{\sqrt{1-x^2}} : x \neq \pm 1\right\}$

$= (-\infty, -1) \cup (1, \infty)$

24. ***(a)*** We have, $[a \quad b \quad c]\begin{bmatrix}1 & 9 & 7\\ 2 & 8 & 7\\ 7 & 3 & 7\end{bmatrix} = [0 \quad 0 \quad 0]$

$\Rightarrow [a + 2b + 7c \quad 9a + 8b + 3c \quad 7a + 7b + 7c] = [0 \ 0 \ 0]$

$\Rightarrow a + 2b + 7c = 0,\ 9a + 8b + 3c = 0$ and $7a + 7b + 7c = 0$

$\Rightarrow a = 1,\ b = 6$ and $c = -7$

Clearly, $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and it satisfies $\omega^3 = 1$, $\frac{1}{\omega} = \omega^2$.

$\therefore \frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega} + \frac{1}{\omega^6} + \frac{3}{\omega^{-7}} = 3\omega^2 + 1 + 3\omega$

$= 3(\omega^2 + \omega) + 1 = -3 + 1 = -2$

25. ***(a)*** We have, $|z - i|z|| = |z + i|z||$

$\Rightarrow \quad \left|\frac{z}{|z|} - i\right| = \left|\frac{z}{|z|} + i\right|$

$\Rightarrow \frac{z}{|z|}$ lies on real axis. $\Rightarrow \text{Im}\left(\frac{z}{|z|}\right) = 0 \Rightarrow \text{Im}(z) = 0$

26. ***(d)*** Given, $z = \cos\theta + i\sin\theta = e^{i\theta}$

$\therefore \sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \text{Im}(e^{i\theta})^{2m-1} = \sum_{m=1}^{15} \text{Im}\, e^{i(2m-1)\theta}$

$= \sin\theta + \sin 3\theta + \sin 5\theta + \ldots + \sin 29\theta$

$= \frac{\sin\left(\frac{\theta + 29\theta}{2}\right)\sin\left(\frac{15 \times 2\theta}{2}\right)}{\sin\left(\frac{2\theta}{2}\right)} = \frac{\sin 15\theta \sin 15\theta}{\sin\theta}$

$= \frac{\sin 30^\circ \sin 30^\circ}{\sin 2^\circ} = \frac{1}{4\sin 2^\circ}$ $\quad [\because \theta = 2^\circ]$

27. ***(c)*** Let $z = |a + b\omega + c\omega^2| \Rightarrow z^2 = |a + b\omega + c\omega^2|^2$

$\Rightarrow \quad z^2 = (a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow \quad z^2 = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$ $\quad \ldots(i)$

Since, a, b, c are all integers but not all simultaneously equal.

If $a = b$, then $a \neq c$ and $b \neq c$

$[\because$ difference of integers = integer$]$

$\Rightarrow (b-c)^2 \geq 1$ $\quad [\because$ minimum difference of two consecutive integers is $\pm$ 1$]$

Also, $(c-a)^2 \geq 1$ and we have taken $a = b \Rightarrow (a-b)^2 = 0$

From Eq. (i), $z^2 \geq \frac{1}{2}(0 + 1 + 1) \Rightarrow z^2 \geq 1$

Hence, minimum value of $|z|$ is 1.

28. ***(b)*** We have, $(1 + \omega)^7 = A + B\omega$

$\Rightarrow \quad (-\omega^2)^7 = A + B\omega \quad \Rightarrow -\omega^{14} = A + B\omega$

$\Rightarrow \quad -\omega^2 = A + B\omega \Rightarrow 1 + \omega = A + B\omega \Rightarrow A = 1, B = 1$

29. ***(b)*** Let $z = x + iy$. Then, $z + 1 = (x + 1) + iy$

$\Rightarrow \quad (z+1)^2 = (x+1)^2 - y^2 + 2i(x+1)y$

$\therefore \quad \text{Re}(z+1)^2 = |z|^2 + 1 \Rightarrow (x+1)^2 - y^2 = x^2 + y^2 + 1$

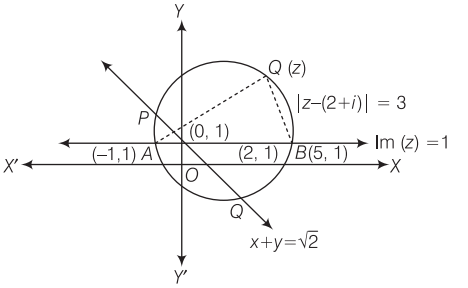
$\Rightarrow \quad x = y^2$, which is parabola.

30. ***(b)*** Let $z = x + iy$

Then, $(1 - i)z = (1 - i)(x + iy) = (x + y) + i(y - x)$

$\Rightarrow \quad \text{Re}|(1 - i)z| = x + y$

$\therefore \quad \text{Re}|(1 - i)z| = \sqrt{2} \Rightarrow x + y = \sqrt{2}$



Clearly, A is the set of all points lying on or above the line $y = 1$ in XY-plane, B is the set of all points lying on the circle having centre at (2, 1) and radius 3 and C is the set of all points on the line $x + y = \sqrt{2}$.

The line $x + y = \sqrt{2}$ intersects the circle $|z - (2 + i)| = 3$ at P and Q out of which P lies in the region represented by $\text{Im}(z) \geq 1$.

$\therefore A \cap B \cap C$ contains exactly one point.

THE POPULAR FOREIGN BOOKS USEFUL FOR JEE ASPIRANTS

CLASSIC TEXT SERIES

C181	A PROBLEM BOOK IN MATHEMATICAL ANALYSIS *GN BERMAN*	180/-
C182	PROBLEMS IN MATHEMATICS *V GOVOROV, P DYBOV, N MIROSHIN, S SMIRNOVA*	160/-
G437	PROBLEMS IN CALCULUS OF ONE VARIABLE *IA MARON*	160/-
C264	HIGHER ALGEBRA *BARNARD & CHILD*	245/-
C048	HIGHER ALGEBRA *HALL & KNIGHT*	195/-
F042	THE ELEMENTS OF STATICS & DYNAMICS PART 1 (STATICS) *SL LONEY*	115/-
F043	THE ELEMENTS OF STATICS & DYNAMICS PART 2 (DYNAMICS) *SL LONEY*	95/-
C047	THE ELEMENTS OF COORDINATE GEOMETRY PART 1 *SL LONEY*	140/-
C046	PLANE TRIGONOMETRY PART 1 *SL LONEY*	95/-
C260	INTEGRAL CALCULUS FOR BEGINNERS *JOSEPH EDWARDS*	95/-
C262	DIFFERENTIAL CALCULUS FOR BEGINNERS *JOSEPH EDWARDS*	95/-
C265	ALGEBRA FOR BEGINNERS *HALL & KNIGHT*	130/-
C266	A SCHOOL GEOMETRY *HS HALL & FH STEVENS*	120/-
C267	ELEMENTARY ALGEBRA FOR SCHOOLS *HS HALL & SR KNIGHT*	195/-
C183	PROBLEMS IN GENERAL PHYSICS *IE IRODOV*	130/-
C261	SCIENCE FOR EVERYONE *SS KROTOV*	90/-
C259	FUNDAMENTAL LAWS OF MECHANICS *IE IRODOV*	105/-
C263	BASIC LAWS OF ELECTROMAGNETISM *IE IRODOV*	125/-

BITSAT Beats

30 Solved Questions for BITSAT 2017

1. In a certain test, 2^{n-1} students gave wrong answers to atleast i questions, where $i = 1, 2, 3, \ldots, k$. If no student gave more than k wrong answers, then the total number of wrong answers given, is

(a) 2^n (b) 2^{n-1}

(c) $2^n - 1$ (d) None of these

2. If $x = 15$ is a root of the equation

$$\log_\pi (x^2 + 15a^2) - \log_\pi (a - 2) = \log_\pi \frac{8ax}{a-2},\ a \neq 2,$$ then

the other root of this equation is

(a) 25 (b) 20 (c) 30 (d) 18

3. If S_n denotes the sum of products of the first n natural number taken two at a time, then $\sum_{n=0}^{\infty} \frac{S_n}{(n+1)!}$ equals

(a) $\frac{11e}{24}$ (b) $\frac{11e}{12}$ (c) $\frac{13e}{24}$ (d) $\frac{13e}{12}$

4. In the binomial expansion $(2^{1/3} + 3^{-1/3})^n$, if the ratio of the seventh term from the beginning of the expansion to the seventh term from its end is 1/6, then n equals

(a) 8 (b) 6 (c) 7 (d) 9

5. Let θ_1 and θ_2 be two angles satisfying $a\cos 2\theta + b\sin 2\theta = c$, then the value of $\cos^2\theta_1 + \cos^2\theta_2$ is

(a) $\frac{a^2 + ac + b^2}{a^2 + b^2}$ (b) $\frac{a^2 + ab + c^2}{a^2 + c^2}$

(c) $\frac{a^2 + bc + c^2}{a^2 + c^2}$ (d) $\frac{a^2 + ab + b^2}{a^2 + b^2}$

6. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$, then $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\theta$ is equal to

(a) $\cos^2\theta$ (b) $\sin^2\theta$ (c) $\cos\theta$ (d) $\sin\theta$

7. A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles is 0.4 and the probability that it contains exactly 3 defective articles is 0.6. Articles are drawn from the lot at random one-by-one without replacement until all the defective are found. The probability that the testing procedure stops at the twelfth testing, is

(a) $\frac{44}{1900}$ (b) $\frac{55}{1900}$ (c) $\frac{77}{1900}$ (d) $\frac{99}{1900}$

8. A variable plane passes through a fixed point (a, b, c) and meets the coordinate axes in A, B, C. The locus of the point common to plane through A, B, C parallel to coordinate planes is

(a) $ayz + bzx + cxy = xyz$ (b) $axy + byz + czx = xyz$

(c) $axy + byz + czx = abc$ (d) $bcx + acy + abz = abc$

9. The plane $ax + by = 0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. The equation of the plane in new position is

(a) $ax + by \pm z\sqrt{a^2 + b^2}\sin\alpha = 0$

(b) $ax + by \pm z\sqrt{a^2 + b^2}\cos\alpha = 0$

(c) $ax + by \pm z\sqrt{a^2 + b^2}\tan\alpha = 0$

(d) None of the above

10. If $\bar{x}$ is the mean and mean deviation from mean is MD $(\bar{x})$, then the number of observations lying between $\bar{x}$ – MD $(\bar{x})$ and $\bar{x}$ + MD $(\bar{x})$ for the data 34, 66, 30, 38, 44, 50, 40, 60, 42, 51, are

(a) 6 (b) 5 (c) 4 (d) 3

11. The area of the region lying inside $x^2 + (y-1)^2 = 1$ and outside $c^2x^2 + y^2 = c^2$, where $c = \sqrt{2} - 1$, is

(a) $\left(\frac{1}{\sqrt{2}} - \frac{2\sqrt{2}+1}{2\sqrt{2}}\pi\right)$ sq units

(b) $\left(\frac{1}{\sqrt{2}} + \frac{2\sqrt{2}+1}{2}\pi\right)$ sq units

(c) $\left(\frac{1}{\sqrt{2}} + \frac{2\sqrt{2}-1}{2}\pi\right)$ sq units

(d) $\left(\frac{1}{\sqrt{2}} - \frac{2\sqrt{2}-1}{2}\pi\right)$ sq units

12. If the first and $(2n-1)$th terms of an AP, GP and HP are equal and their nth terms are x, y and z respectively, then

(a) $x + z = 2y$ (b) $x + z = y$

(c) $xz - y^2 = 0$ (d) None of these

13. The range of $\sqrt{\ln\left[\frac{1}{|\sin x + \cos x|}\right]}$ is

(a) $[1, \infty)$ (b) $\{0\}$

(c) $(0, 1]$ (d) $(-\infty, 1]$

14. The particular solution of differential equation $(1 + \cos 2x)\frac{dy}{dx} - (1 + e^{y})\sin 2x = 0$; given that $y = 0$, when $x = \frac{\pi}{4}$, is

(a) $2\sqrt{2}\cos x = e^{y} + 1$ (b) $2\sqrt{2}\cos x = e^{y} - 1$
(c) $2\sqrt{2}\cos x = e^{-y} + 1$ (d) $2\sqrt{2}\cos x = e^{-y} - 1$

15. If $\mathbf{r}$, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are the non-zero vectors such that $\mathbf{r}\cdot\mathbf{a} = \mathbf{r}\cdot\mathbf{b} = \mathbf{r}\cdot\mathbf{c} = 0$, then $[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]$

(a) is equal to 1 (b) is equal to zero
(c) Cannot be determined (d) None of these

16. The value of $\int_{\alpha}^{\beta}\sqrt{(x-\alpha)(\beta-x)}dx$, $\alpha < \beta$, is

(a) $\frac{1}{8}\pi(\beta - \alpha)^3$ (b) $\frac{1}{8}\pi(\beta + \alpha)^3$
(c) $\frac{1}{8}\pi(\beta - \alpha)^2$ (d) $\frac{1}{8}\pi(\beta + \alpha)^2$

17. If $\lim_{x\to 0}\left[\frac{axe^{x} - b\log(1+x) + cxe^{-x}}{x^2\sin x}\right] = 2$, then the value of (a, b, c) is

(a) (12, 9, 3) (b) (3, 9, 12) (c) (9, 3, 12) (d) (3, 12, 9)

18. If $f(x) = \lim_{n\to\infty}\frac{\log(x+2) - x^{2n}\sin x}{x^{2n} + 1}$, then $f(x)$ is

(a) continuous at $x = \pm 1$ (b) discontinuous at $x = \pm 1$
(c) continuous at $x = \pm 2$ (d) discontinuous at $x = \pm 2$

19. If $f(x) = [\sqrt{2}\sin x]$, where $[x]$ denotes the greatest integer function, then

(a) $f(x)$ is continuous at $x = 0$
(b) maximum value of $f(x)$ is 1 in $[-2\pi, 2\pi]$
(c) $f(x)$ is discontinuous at $x = \frac{n\pi}{2} + \frac{\pi}{4}$, $n \in I$
(d) $f(x)$ is differentiable at $x = n\pi$, $n \in I$

20. If $\int\frac{\cos x\,dx}{\sin^2 x(\sin x + \cos x)} = A - \ln|\sin x| + B + k$, then the value of (A, B) is

(a) $(\ln|\cos x|, \cot x)$ (b) $(\ln|\sin x - \cos x|, \tan x)$
(c) $(\ln|\sin x + \cos x|, -\cot x)$ (d) $(\ln|\sin x + \cos x|, -\tan x)$

21. A window of fixed perimeter (including the base of the arch) is in the form of a rectangle surmounted by a semicircle. The semicircle portion is fitted with colour glass while the rectangular part is fitted with clear glass. The clear glass transmits three times as much light per square metre as the coloured glass does. The ratio of the sides of the rectangle so that the window transmits maximum light, is

(a) $\frac{6}{6+\pi}$ (b) $\frac{6}{6-\pi}$ (c) $\frac{6+\pi}{6}$ (d) $\frac{6-\pi}{6}$

22. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$, then $f(100)$ equals

(a) 0 (b) 1 (c) 100 (d) -100

23. The nature of solutions of the system of the equations $x + 3y - 2z = 0$, $2x - y + 4z = 0$ and $x - 11y + 14z = 0$, is

(a) consistent (b) infinite
(c) unique (d) None of these

24. $PQRS$ is a rhombus. Its diagonals PR and QS intersect at O such that $QS = 2PR$. If the points S and O represent the complex numbers $(1 + i)$ and $(2 - i)$ respectively, then the complex number(s) representing P, is

(a) $1 - \frac{3i}{2}$ (b) $3 - \frac{1}{2}i$
(c) Both (a) and (b) (d) None of these

25. The equation of the circle whose diameter is the common chord of the circles $x^2 + y^2 + 3x + 2y + 1 = 0$ and $x^2 + y^2 + 3x + 4y + 2 = 0$, is

(a) $x^2 + y^2 + 8x + 10y + 2 = 0$
(b) $x^2 + y^2 - 5x + 4y + 7 = 0$
(c) $2x^2 + 2y^2 + 6x + 2y + 1 = 0$
(d) None of the above

26. The locus of a point P which moves such that two of the three normals drawn from it to the parabola $y^2 = 4ax$ are mutually perpendicular, is

(a) $y^2 = a(x + 3a)$ (b) $y^2 = a(x - 3a)$
(c) $y^2 = a(x + 2a)$ (d) $y^2 = a(x - 2a)$

27. If PS_1Q and PS_2R are two focal chords of the ellipse whose two foci are S_1 and S_2 and the eccentric angle of P is θ, then the equation of chord QR is $\frac{x}{a}A + \frac{y}{b}B\sin\theta + C = 0$. The value of (A, B, C) is

(a) $\left(\cos\theta, \frac{1+e^2}{1-e^2}, 2\right)$ (b) $\left(\cos\theta, \frac{1-e^2}{1+e^2}, 2\right)$
(c) $\left(\cos\theta, \frac{1+e^2}{1-e^2}, 1\right)$ (d) $\left(\cos\theta, \frac{1-e^2}{1+e^2}, 1\right)$

28. If a rectangular hyperbola has the equation $xy = c^2$, then the locus of the middle points of its chords of length $2d$, is

(a) $(x^2 + y^2)(xy + c^2) = d^2xy$
(b) $(x^2 - y^2)(xy - c^2) = d^2xy$
(c) $(x^2 + y^2)(xy - c^2) = d^2xy$
(d) None of the above

29. A line $4x + y = 1$ through the point $A(2, -7)$ meets the line BC whose equation is $3x - 4y + 1 = 0$ at the point B. If $AB = AC$, then the equation of the line AC is

(a) $89y + 52x + 519 = 0$ (b) $89y + 52x - 519 = 0$
(c) $52x + 89y - 519 = 0$ (d) $52x + 89y + 519 = 0$

30. If $\sim(q \to p)$ is true, then the truth value of p and q are respectively

(a) (F, F) (b) (F, T)
(c) (T, F) (d) None of these

Answers with Explanation

1. ***(c)*** The number of answering exactly $k\ (1 \le k \le n-1)$ questions wrongly is $2^{n-k} - 2^{n-k-1}$.
The number of students answering all questions wrongly is 2^0.
Hence, the total number of wrong answers

$$= 1\,(2^{n-1} - 2^{n-2}) + 2\,(2^{n-2} - 2^{n-3}) + \ldots + (n-1)\,(2^1 - 2^0) + 2^0 \cdot n$$
$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots + 2^1 + 2^0 = 2^n - 1$$

2. ***(a)*** Given equation is

$$\log_\pi(x^2 + 15a^2) - \log_\pi(a-2) = \log_\pi \frac{8ax}{a-2},\ a \ne 2$$

$$\Rightarrow \log_\pi(x^2 + 15a^2) - \log_\pi(a-2) = \log_\pi 8ax - \log_\pi(a-2)$$
$$\Rightarrow \log_\pi(x^2 + 15a^2) = \log_\pi 8ax$$
$$\Rightarrow x^2 - 8ax + 15a^2 = 0$$
$$\Rightarrow (x-5a)(x-3a) = 0$$
$$\Rightarrow x = 5a, 3a$$

But $x = 15$ is given to be a root of the equation.
Hence, other roots of the equation are 25 and 9 accordingly as $a = 5$ and $a = 3$.

3. ***(a)*** Here, $S_n = \frac{1}{2}[(1 + 2 + \ldots + n)^2 - (1^2 + 2^2 + \ldots + n^2)]$

$$\left[\because \Sigma ab = \frac{1}{2}\{(\Sigma a)^2 - \Sigma a^2\}\right]$$

$$= \frac{1}{2}\left[\left\{\frac{n(n+1)}{2}\right\}^2 - \frac{n(n+1)(2n+1)}{6}\right]$$
$$= \frac{1}{24}n(n-1)(n+1)(3n+2)$$

$$\therefore \sum_{n=1}^{\infty} \frac{S_n}{(n+1)!} = \frac{1}{24}\sum_{n=1}^{\infty} \frac{n(n-1)(n+1)(3n+2)}{(n+1)!}$$
$$= \frac{1}{24}\sum_{n=1}^{\infty} \frac{n(n-1)(3n+2)}{n!}$$
$$= \frac{1}{24}\sum_{n=1}^{\infty} \frac{3n^3 - n^2 - 2n}{n!}$$
$$= \frac{1}{24}\left[3\sum_{n=1}^{\infty}\frac{n^3}{n!} - \sum_{n=1}^{\infty}\frac{n^2}{n!} - 2\sum_{n=1}^{\infty}\frac{n}{n!}\right]$$
$$= \frac{1}{24}[3 \times 5e - 2e - 2e] = \frac{11e}{24}$$

4. ***(d)*** We have, T_{r+1} in $(2^{1/3} + 3^{-1/3})^n$

$$= {}^nC_r\, 2^{\frac{n-r}{3}}\, 3^{-r/3}$$

$$\Rightarrow \frac{\text{7th term from beginning}}{\text{7th term from end}} = \frac{{}^nC_6\, 2^{\frac{n-6}{3}}\, 3^{6/3}}{{}^nC_6 (3^{-1})^{\frac{n-6}{3}}\, 2^{6/3}}$$
$$= 2^{\frac{n-12}{3}} \cdot 3^{\frac{n-12}{3}}$$

$$\Rightarrow 6^{-1} = 6^{\frac{n-12}{3}} \Rightarrow -1 = \frac{n-12}{3}$$
$$\Rightarrow n = 9$$

5. ***(a)*** Given, $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow b\sin 2\theta = c - a\cos 2\theta$$
$$\Rightarrow b^2\sin^2 2\theta = c^2 - 2ac\cos 2\theta + a^2\cos^2 2\theta$$
$$\Rightarrow b^2(1 - \cos^2 2\theta) = c^2 - 2ac\cos 2\theta + a^2\cos^2 2\theta$$
$$\Rightarrow (a^2 + b^2)\cos^2 2\theta - 2ac\cos 2\theta + c^2 - b^2 = 0$$

Let $\cos 2\theta_1$ and $\cos 2\theta_2$ be the roots of the above equation.

$$\therefore \text{Sum of roots} = \cos 2\theta_1 + \cos 2\theta_2 = \frac{2ac}{a^2 + b^2}$$

$$\Rightarrow (2\cos^2\theta_1 - 1) + (2\cos^2\theta_2 - 1) = \frac{2ac}{a^2 + b^2}$$
$$\Rightarrow 2(\cos^2\theta_1 + \cos^2\theta_2) = \frac{2ac}{a^2 + b^2} + 2$$
$$\Rightarrow 2(\cos^2\theta_1 + \cos^2\theta_2) = \frac{2ac + 2a^2 + b^2}{a^2 + b^2}$$
$$\Rightarrow \cos^2\theta_1 + \cos^2\theta_2 = \frac{a^2 + ac + b^2}{a^2 + b^2}$$

6. ***(b)*** Given, $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$

$$\Rightarrow \cos^{-1}\left(\frac{x}{a}\cdot\frac{y}{b} - \sqrt{1 - \frac{x^2}{a^2}}\sqrt{1 - \frac{y^2}{b^2}}\right) = \theta$$
$$\Rightarrow \frac{xy}{ab} - \sqrt{\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)} = \cos\theta$$
$$\Rightarrow \frac{xy}{ab} - \cos\theta = \sqrt{\left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)}$$
$$\Rightarrow \left(\frac{xy}{ab} - \cos\theta\right)^2 = \left(1 - \frac{x^2}{a^2}\right)\left(1 - \frac{y^2}{b^2}\right)$$
$$\Rightarrow \frac{x^2y^2}{a^2b^2} - \frac{2xy}{ab}\cos\theta + \cos^2\theta = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2}$$
$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\theta = \sin^2\theta$$

7. ***(d)*** Suppose A is the event that the testing procedure ends at the twelfth testing, A_1 is the event that the lot contains 2 defective and A_2 is the event that the lot contains 3 defective.
Then, $P(A_1) = 0.4$ and $P(A_2) = 0.6$

$$\therefore P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2)$$
$$= 0.4\left(\frac{{}^{18}C_{10}\cdot{}^2C_1}{{}^{20}C_{11}}\cdot\frac{1}{9}\right) + 0.6\left(\frac{{}^{17}C_9\cdot{}^3C_2}{{}^{20}C_{11}}\cdot\frac{1}{9}\right)$$
$$= \frac{1}{9}\left[\frac{4}{10}\cdot\frac{18!}{10!\,18!}\cdot\frac{11!\,9!}{20!}\cdot 2 + \frac{6}{10}\cdot\frac{7!}{9!\,18!}\cdot\frac{11!\,9!}{20!}\cdot 3\right]$$
$$= \frac{1}{9}\left[\frac{4}{10}\times\frac{11\times 9}{19\times 20}\times 2 + \frac{6}{10}\times\frac{9\times 10\times 11}{18\times 19\times 20}\times 3\right]$$
$$= \frac{44}{1900} + \frac{55}{1900} = \frac{99}{1900}$$

8. ***(a)*** Let the equation of the plane be $\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$.

$\Rightarrow \quad \frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=1$

[since, the plane passes through a, b, c]

Now, the points of intersection of the plane with the coordinate axes are $A(\alpha, 0, 0)$, $B(0, \beta, 0)$ and $C(0, 0, \gamma)$.
Then, equation of planes parallel to the coordinate planes and passing through A, B and C, are $x=\alpha$, $\gamma=\beta$ and $z=\gamma$.
$\therefore$ The locus of the common point is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$$

[by eliminating α, β, γ from the above equation]

$\Rightarrow \quad ayz+bzx+cxy=xyz$

9. ***(c)*** Given planes are $ax+by=0$ ...(i)

and $z=0$ ...(ii)

Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as

$$ax+by+kz=0 \quad \text{...(iii)}$$

The direction cosines of the normal to the plane (iii) are

$$\frac{a}{\sqrt{a^2+b^2+k^2}}, \frac{b}{\sqrt{a^2+b^2+k^2}}, \frac{c}{\sqrt{a^2+b^2+k^2}}$$

The direction cosines of the normal to the plane (i) are

$$\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, 0$$

Since, the angle between planes (i) and (iii) is α.

$\therefore \quad \cos\alpha=\frac{a\cdot a+b\cdot b+k\cdot 0}{\sqrt{a^2+b^2+k^2}\cdot\sqrt{a^2+b^2}}$

$\Rightarrow \quad \cos\alpha=\sqrt{\frac{a^2+b^2}{a^2+b^2+k^2}}$

$\Rightarrow \quad (a^2+b^2+k^2)\cos^2\alpha=a^2+b^2$

$\Rightarrow \quad k^2\cos^2\alpha=a^2(1-\cos^2\alpha)+b^2(1-\cos^2\alpha)$

$\Rightarrow \quad k^2=\frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}$

$\Rightarrow \quad k=\pm\sqrt{a^2+b^2}\tan\alpha$

On putting the value of k in Eq. (iii), we get

$$ax+by\pm z\sqrt{a^2+b^2}\tan\alpha=0$$

10. ***(a)*** Given data can be arranged in ascending order as
30, 34, 38, 40, 42, 44, 50, 51, 60, 66

$\therefore \quad \bar{x}=\frac{30+34+38+40+42+44+50+51+60+66}{10}$

$=\frac{455}{10}=45.5$

Now,
$|x_i-\bar{x}|=15.5, 11.5, 7.5, 5.5, 3.5, 1.5, 4.5, 5.5, 14.5, 20.5$

Mean deviation from the mean $=\text{MD}(\bar{x})=\frac{\Sigma|(x_i-\bar{x})|}{10}$

$=\frac{1}{10}(15.5+11.5+7.5+5.5+3.5+1.5+4.5+5.5+14.5+20.5)$

$=\frac{1}{10}(90.0)=9$

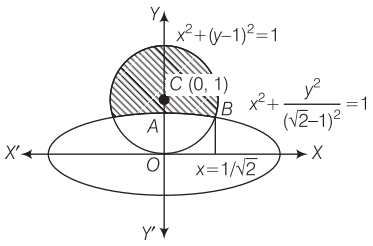
Now, $\bar{x}-\text{MD}(\bar{x})=45.5-9=36.5$

and $\bar{x}+\text{MD}(\bar{x})=45.5+9=54.5$

Given observations which lie between 36.5 and 54.5, are 38, 40, 42, 44, 50, 51, which are six in number.

11. ***(c)*** Given curves are $x^2+(y-1)^2=1$ ...(i)

and $x^2+\frac{y^2}{(\sqrt{2}-1)^2}=1$ ...(ii)



On subtracting Eq. (ii) from Eq. (i), we get

$$(y-1)^2-\frac{y^2}{(\sqrt{2}-1)^2}=0$$

$\Rightarrow \quad y-1=\pm\frac{y}{\sqrt{2}-1}$

$\Rightarrow \quad y-1=\pm(\sqrt{2}+1)\,y$

$\Rightarrow \quad \sqrt{2}y=-1 \text{ or } (2+\sqrt{2})\,y=1$

$\Rightarrow \quad y=\frac{-1}{\sqrt{2}} \text{ or } y=\frac{1}{2+\sqrt{2}}$

$\therefore \quad y=\frac{1}{2+\sqrt{2}}$ [$\because y>0$ for point B]

For $y=\frac{1}{2+\sqrt{2}}$,

$x^2=1-\frac{y^2}{(\sqrt{2}-1)^2}$ [from Eq. (ii)]

$\Rightarrow \quad x^2=1-\frac{1}{2(\sqrt{2}+1)^2(\sqrt{2}-1)^2}$

$\Rightarrow \quad x^2=1-\frac{1}{2}=\frac{1}{2} \Rightarrow x=\pm\frac{1}{\sqrt{2}}$

$\Rightarrow \quad x=\frac{1}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}}$

$\therefore \quad x=\frac{1}{\sqrt{2}}$ [$\because x>0$ for point B]

Now, required area $=\pi(1)^2-2\times$ Area of $OABO$

$=\pi-2\int_0^{1/\sqrt{2}}(y_1-y_2)dx$

$=\pi-2(\sqrt{2}-1)\int_0^{1/\sqrt{2}}\sqrt{1-x^2}dx+2\int_0^{1/\sqrt{2}}(1-\sqrt{1-x^2})\,dx$

$=\pi-2(\sqrt{2}-1)\left[\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x\right]_0^{1/\sqrt{2}}+2[x]_0^{1/\sqrt{2}}-2\left[\frac{x\sqrt{1-x^2}}{2}+\frac{1}{2}\sin^{-1}x\right]_0^{1/\sqrt{2}}$

$=\pi-2(\sqrt{2}-1)\left(\frac{1}{4}+\frac{1}{2}\cdot\frac{\pi}{4}\right)+2\left(\frac{1}{\sqrt{2}}-0\right)-2\left(\frac{1}{4}+\frac{1}{2}\cdot\frac{\pi}{4}\right)$

$=\pi-2\sqrt{2}\left(\frac{2+\pi}{8}\right)+\sqrt{2}=\pi-\frac{1}{\sqrt{2}}-\frac{2\sqrt{2}\pi}{8}+\sqrt{2}$

$=\left(\frac{1}{\sqrt{2}}+\frac{2\sqrt{2}-1}{2\sqrt{2}}\pi\right)$ sq units

12. ***(c)*** Let α be the first and β be the $(2n-1)$th terms of an AP, GP and HP, then α, x, β will be in AP; α, y, β will be in GP and α, z, β will be in HP.

Hence, x, y and z are respectively AM, GM and HM of α and β.

$$\therefore \quad x = \frac{\alpha+\beta}{2},\ y^2 = \alpha\beta \text{ and } z = \frac{2\alpha\beta}{\alpha+\beta}$$

Hence, $xz - y^2 = 0$

13. ***(b)*** We know that, $|\sin x| + |\cos x|$ varies from 1 to $\sqrt{2}$.

$\Rightarrow$ Range of $\dfrac{1}{|\sin x| + |\cos x|}$ is $\left[\dfrac{1}{\sqrt{2}}, 1\right]$.

$\Rightarrow$ Range of $\ln\left(\dfrac{1}{|\sin x| + |\cos x|}\right)$ is $\left[\ln\dfrac{1}{\sqrt{2}}, 0\right]$.

Also, $\ln\dfrac{1}{\sqrt{2}} < 0$

$\Rightarrow$ Range of $\sqrt{\ln\left(\dfrac{1}{|\sin x| + |\cos x|}\right)}$ has only value equal to zero.

Hence, range of $\sqrt{\ln\left(\dfrac{1}{|\sin x| + |\cos x|}\right)}$ is $\{0\}$.

14. ***(c)*** Given, $(1 + \cos 2x)\dfrac{dy}{dx} - (1 + e^y)\sin 2x = 0$

$$\Rightarrow \quad \frac{dy}{1+e^y} = \frac{\sin 2x}{1+\cos 2x}dx \text{ [on separating the variables]}$$

$$\Rightarrow \quad \int\left[\frac{(1+e^y)-e^y}{1+e^y}\right]dy = \int \tan x\ dx$$

$$\Rightarrow \quad y - \log(1+e^y) + \log\cos x = A$$

$$\Rightarrow \quad y + \log\left(\frac{\cos x}{1+e^y}\right) = A$$

On taking $y = 0$, when $x = \dfrac{\pi}{4}$, we get

$$0 + \log\left(\frac{1}{2\sqrt{2}}\right) = A \ \Rightarrow\ A = \frac{-3}{2}\log 2$$

$\therefore$ The particular solution is

$$y + \log\left[\left(\frac{\cos x}{1+e^y}\right)2\sqrt{2}\right] = 0$$

$$\Rightarrow \quad 2\sqrt{2}\cos x = (1+e^y)\,e^{-y}$$

$$\Rightarrow \quad 2\sqrt{2}\cos x = e^{-y} + 1$$

15. ***(b)*** Since, $\mathbf{r}\cdot\mathbf{a} = 0, \mathbf{r}\cdot\mathbf{b} = 0$ and $\mathbf{r}\cdot\mathbf{c} = 0$

Therefore, $\mathbf{r}$ must be perpendicular to all the three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Hence, $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ are coplanar.

$$\therefore \quad [\mathbf{a}\ \mathbf{b}\ \mathbf{c}] = 0$$

16. ***(c)*** Let $I = \int_\alpha^\beta \sqrt{(x-\alpha)(\beta-x)}\,dx,\ \alpha < \beta$ ...(i)

Now, $(x-\alpha)(\beta-x) = -[x^2 - (\alpha+\beta)x + \alpha\beta]$

$$= -\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 + \alpha\beta - \left(\frac{\alpha+\beta}{2}\right)^2\right]$$

$$= -\left[\left(x - \frac{\alpha+\beta}{2}\right)^2 - \left(\frac{\alpha-\beta}{2}\right)^2\right]$$

$$\Rightarrow (x-\alpha)(\beta-x) = \left(\frac{\alpha-\beta}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2$$

On putting this value in Eq. (i), we get

$$I = \int_\alpha^\beta \sqrt{\left(\frac{\alpha-\beta}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2}\,dx$$

$$= \left[\frac{1}{2}\left(x - \frac{\alpha+\beta}{2}\right)\sqrt{\left(\frac{\alpha-\beta}{2}\right)^2 - \left(x - \frac{\alpha+\beta}{2}\right)^2}\right.$$

$$\left. + \frac{1}{8}(\beta-\alpha)^2\sin^{-1}\left(\frac{x - \frac{\alpha+\beta}{2}}{\frac{\beta-\alpha}{2}}\right)\right]_\alpha^\beta$$

$$= \left[0 + \frac{1}{8}(\beta-\alpha)^2\sin^{-1}(1)\right] - \left[0 + \frac{1}{8}(\beta-\alpha)^2\sin^{-1}(-1)\right]$$

$$= 2\cdot\frac{1}{8}(\beta-\alpha)^2\left(\frac{\pi}{2}\right)$$

$$= \frac{\pi}{8}(\beta-\alpha)^2$$

17. ***(d)*** Using expansion, we have

$$\lim_{x\to 0} ax\left(1 + x + \frac{x^2}{2!} + \ldots\right) - b\left(x - \frac{x^2}{2} + \frac{x^2}{3} - \ldots\right) + cx\left(1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} - \ldots\right) \Big/ x^2\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right) = 2$$

$$\Rightarrow \lim_{x\to 0} \frac{x(a-b+c) + x^2\left(a + \frac{b}{2} - c\right) + x^3\left(\frac{a}{2} - \frac{b}{3} + \frac{c}{2}\right) + \ldots}{x^2 - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\right)} = 2$$

Now, above limit would exist, if least power in numerator is greater than or equal to least power in denominator, i.e. coefficient of x or x^2 must be zero and coefficient of x^3 should be 2.

$$\therefore a - b + c = 0,\ a + \frac{b}{2} - c = 0 \text{ and } \frac{a}{2} - \frac{b}{3} + \frac{c}{2} = 2$$

On solving, we get

$$a = 3,\ b = 12 \text{ and } c = 9$$

$$\therefore (a, b, c) = (3, 12, 9)$$

18. ***(b)*** If $|x| < 1$, then $f(x) = \lim_{n\to\infty} \dfrac{\log(x+2) - x^{2n}\sin x}{x^{2n}+1}$

$$= \frac{\log(x+2)}{1} = \log(x+2)$$

If $|x| > 1$, then $f(x) = \lim_{n\to\infty} \dfrac{\log(x+2) - x^{2n}\sin x}{x^{2n}+1}$

$$= \lim_{n\to\infty} \frac{\frac{\log(x+2)}{x^{2n}} - \sin x}{1 + \frac{1}{x^{2n}}} = -\sin x$$

If $|x| = 0$, then $f(x) = \lim_{n\to\infty} \dfrac{\log(x+2) - x^{2n}\sin x}{x^{2n}+1}$

$= \dfrac{\log(x+2) - \sin x}{2}$

Thus, $f(x) = \begin{cases} -\sin x, & x < -1 \\ \log(x+2), & -1 < x < 1 \\ -\sin x, & x > 1 \\ \dfrac{\log(x+2) - \sin x}{2}, & x = \pm 1 \end{cases}$

Hence, $f(x)$ is discontinuous at $x = \pm 1$.

19. ***(b)*** Given, $f(x) = [\sqrt{2}\sin x]$

$\therefore \quad f(x) = -2, -\dfrac{\pi}{2} \le x < -\dfrac{\pi}{4} = -1, -\dfrac{\pi}{4} \le x < 0 = 0,$

$0 \le x < \dfrac{\pi}{4} = 1, \dfrac{\pi}{4} \le x < \dfrac{\pi}{2} = 1,$

$\dfrac{\pi}{2} \le x \le \dfrac{3\pi}{4} = 0, \dfrac{3\pi}{4} < x \le \pi$

Clearly, $f(x)$ is discontinuous at $x = -\dfrac{\pi}{4}, 0, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$, etc.

General value corresponding to

$x = -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ is $\dfrac{n\pi}{2} + \dfrac{\pi}{4}$.

Maximum value of $f(x)$ in $[-2\pi, 2\pi]$ is 1 at $x = \dfrac{\pi}{2}$.

Therefore, $f(x)$ is discontinuous and non-differentiable at $x = 0$.

20. ***(c)*** Let $I = \int \dfrac{\cos x \sec^2 x}{\sec^2 x \sin^2 x (\sin x + \cos x)}\,dx$

$= \int \dfrac{\sec^2 x}{\tan^2 x (1 + \tan x)}\,dx$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt$

$\therefore \quad I = \int \dfrac{dt}{t^2(1+t)}$

Let $\dfrac{1}{t^2(1+t)} = \dfrac{A}{t} + \dfrac{B}{t^2} + \dfrac{C}{1+t}$

$\Rightarrow \quad 1 = At(1+t) + B(1+t) + Ct^2$

Put $t = 0$, we get $B = 1$; put $t = -1$, we get $C = 1$

Compare the coefficients of t^2, we get $0 = A + C$

$\Rightarrow \quad A = -1$

$\therefore \quad I = -\int \dfrac{dt}{t} + \int \dfrac{dt}{t^2} + \int \dfrac{dt}{1+t}$

$= -\ln|t| - \dfrac{1}{t} + \ln|1+t| + k$

$= \ln\left|\dfrac{1+\tan x}{\tan x}\right| - \cot x + k$

$= \ln\left|\dfrac{1 + \dfrac{\sin x}{\cos x}}{\dfrac{\sin x}{\cos x}}\right| - \cot x + k$

$= \ln\left|\dfrac{\cos x + \sin x}{\sin x}\right| - \cot x + k$

$\Rightarrow I = \ln|\sin x + \cos x| - \ln|\sin x| - \cot x + k$

Hence, $(A, B) = (\ln|\sin x + \cos x|, -\cot x)$

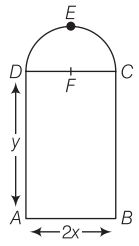
21. ***(a)*** Let $2x$ and y be the dimensions of the rectangular portion.

Then, the radius of the semi-circular portion is x.

Perimeter $= 2x + 2x + 2y + \pi x = k$

$\therefore \quad y = \dfrac{k - x(4+\pi)}{2}$

Let a units of light per square unit area of colour glass be transmitted. Then, $3a$ units will be transmitted by clear glass.



Amount of light (L) transmitted

$= \left(\dfrac{\pi x^2}{2}\right)a + 2xy(3a)$

$= \dfrac{a\pi}{2}x^2 + 6ax\left[\dfrac{k - x(4+\pi)}{2}\right]$

$\Rightarrow \quad \dfrac{dL}{dx} = a\pi x + 3ak - 6ax(4+\pi)$

$\Rightarrow \quad \dfrac{d^2L}{dx^2} = a\pi - 6a(4+\pi)$

Now, $\dfrac{dL}{dx} = 0$, when $x = \dfrac{3ak}{24a + 5a\pi} = \dfrac{3k}{24 + 5\pi}$

$\Rightarrow \quad \dfrac{d^2L}{dx^2} = -a(12 + 5\pi) = \text{Negative}$

$\therefore L$ is maximum when $x = \dfrac{3k}{24 + 5\pi}$.

Hence, ratio of the sides of the rectangle $= \dfrac{2x}{y}$

$= \dfrac{\dfrac{6k}{24+5\pi}}{k - \dfrac{3k(4+\pi)}{24+5\pi}} = \dfrac{12k}{12k + 2k\pi} = \dfrac{6}{6+\pi}$

22. ***(a)*** $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & (x+1)x(x-1) \end{vmatrix}$

On applying $C_3 \to C_3 - (C_1 + C_2)$, we get

$f(x) = \begin{vmatrix} 1 & x & 0 \\ 2x & x(x-1) & 0 \\ 3x(x-1) & x(x-1)(x-2) & 0 \end{vmatrix} = 0$

Hence, $f(100) = 0$

23. ***(b)*** Let $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$

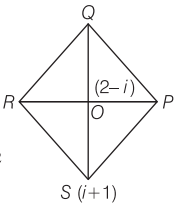
$\Rightarrow |A| = 1(-14 + 44) - 3(28 - 4) - 2(-22 + 1)$

$= 30 - 72 + 42 = 72 - 72 = 0$

$\Rightarrow \quad |A| = 0$

Hence, infinite number of solutions exist.

24. *(c)* Let P be z. The position OP can be obtained by rotating OP anti-clockwise through an angle $\frac{\pi}{2}$, simultaneously length gets halved.



$$\therefore\ z-(2-i)=\frac{1}{2}[(1+i)-(2-i)]e^{i\pi/2}$$

$$=\frac{1}{2}(-2-i)=-1-\frac{1}{2}i$$

$$\Rightarrow\quad z=-1-\frac{1}{2}i+2-i=1-\frac{3i}{2}$$

Another position of P corresponds to P and R getting interchanged and in that the complex number of P is $1+\frac{1}{2}i+2-i=3-\frac{1}{2}i$.

Hence, the complex number of P is either $1-\frac{3i}{2}$ or $3-\frac{1}{2}i$.

25. *(c)* The equation of the common chord of the circles

$$x^2+y^2+3x+2y+1=0$$

and $x^2+y^2+3x+4y+2=0$, is $2y+1=0$

[using $S_1-S_2=0$]

The equation of a circle passing through the intersection of $x^2+y^2+3x+2y+1=0$ and $x^2+y^2+3x+4y+2=0$, is

$$(x^2+y^2+3x+2y+1)+\lambda(x^2+y^2+3x+4y+2)=0$$

$$\Rightarrow\quad x^2+y^2+x\left(\frac{3+3\lambda}{\lambda+1}\right)+2y\left(\frac{1+2\lambda}{\lambda+1}\right)+\left(\frac{1+2\lambda}{\lambda+1}\right)=0 \qquad \ldots(i)$$

Since, $2y+1=0$ is a diameter of this circle, therefore its centre $\left(-\frac{3+3\lambda}{2},-\frac{2\lambda+1}{\lambda+1}\right)$ lies on $2y+1=0$.

$$\therefore\quad -2\left(\frac{2\lambda+1}{\lambda+1}\right)+1=0$$

$$\Rightarrow\quad -4\lambda-2+\lambda+1=0$$

$$\Rightarrow\quad -3\lambda=1$$

$$\Rightarrow\quad \lambda=-1/3$$

On putting $\lambda=-1/3$ in Eq. (i), we get

$$2x^2+2y^2+6x+2y+1=0$$

which is the equation of required circle.

26. *(b)* Let point $P(h, k)$ and normal to parabola be $y=mx-2am-am^3$. If this normal passes through (h, k), then $am^3+(2a-h)m+k=0$ ...(i)

Let m_1, m_2 and m_3 be the roots of Eq. (i), then

$$m_1+m_2+m_3=0 \qquad \ldots(ii)$$

$$m_1m_2+m_1m_3+m_2m_3=\frac{2a-h}{a} \qquad \ldots(iii)$$

and $$m_1m_2m_3=\frac{k}{a} \qquad \ldots(iv)$$

Here, $m_1m_2=-1$ [given]

So, $m_3=\frac{k}{a}$ [from Eq. (iv)]

$\Rightarrow\quad m_1+m_2=-k/a$ [from Eq. (ii)]

On putting these values in Eq. (iii), we get

$$-1-\frac{k^2}{a^2}=\frac{2a-h}{a}\ \Rightarrow\ k^2=a(h-3a)$$

So, locus of P is $y^2=a(x-3a)$.

27. *(c)* Let Q be $(a\cos\alpha, b\sin\alpha)$ and R be $(a\cos\beta, b\sin\beta)$, then the equation of the chord QR is

$$\frac{x}{a}\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)$$

On simplifying, it becomes

$$\frac{x}{a}\left(1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)+\frac{y}{b}\left(\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}\right)=1+\tan\frac{\alpha}{2}\tan\frac{\beta}{2} \qquad \ldots(i)$$

Also, PQ and PR are focal chords.

$$\therefore\quad \tan\frac{\theta}{2}\tan\frac{\alpha}{2}=\frac{e-1}{e+1}$$

and $$\tan\frac{\theta}{2}\tan\frac{\beta}{2}=\frac{e+1}{e-1}$$

On substituting the values of $\tan\frac{\alpha}{2}$ and $\tan\frac{\beta}{2}$ in Eq. (i), we get

$$\frac{x}{a}\left(1-\frac{1}{\tan^2\theta/2}\right)+\frac{y}{a}\left[\frac{e-1}{(e+1)\tan\theta/2}+\frac{e+1}{(e-1)\tan\theta/2}\right]=1+\frac{1}{\tan^2\theta/2}$$

$$\Rightarrow -\frac{x}{a}\left(\frac{1-\tan^2\theta/2}{\tan^2\theta/2}\right)+\frac{y}{b\tan\theta/2}\left[\frac{2(e^2+1)}{e^2-1}\right]\left(\frac{1+\tan^2\theta/2}{\tan^2\theta/2}\right)=0$$

$$\Rightarrow \frac{x}{a}\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right)+\frac{y}{b}\left(\frac{1+e^2}{1-e^2}\right)\frac{2\tan\theta/2}{1+\tan^2\theta/2}+1=0$$

$$\Rightarrow \frac{x}{a}\cos\theta+\frac{y}{b}\left(\frac{1+e^2}{1-e^2}\right)\sin\theta+1=0$$

Hence, value of $(A, B, C)=\left(\cos\theta, \frac{1+e^2}{1-e^2}, 1\right)$

28. *(c)* Let (x', y') be the middle point of the chord of the hyperbola $xy=c^2$ which is inclined to the X-axis at an angle θ. Then, the extremities of the chord, which are at distance d from (x, y), is given by

$$\frac{x-x'}{\cos\theta}=\frac{y-y'}{\sin\theta}=\pm d$$

$$\Rightarrow\quad x=x'\pm d\cos\theta$$

and $$y=y'\pm d\sin\theta$$

Since, these lie on the hyperbola $xy=c^2$.

$$\therefore\quad (x'+d\cos\theta)(y'+d\sin\theta)=c^2$$

and $$(x'-d\cos\theta)(y'-d\sin\theta)=c^2$$

By addition and subtraction, we get

$x'y'+d^2\cos\theta\sin\theta=c^2$ and $x'\sin\theta+y'\cos\theta=0$

On dividing these by $\cos^2\theta$ and $\cos\theta$ respectively, we get

$(x'y'-c^2)\sec^2\theta+d^2\tan\theta=0$ and $\tan\theta=-y'/x'$

On eliminating $\tan\theta$ between these equations and replacing x' by x and y' by y, we get

$$(xy-c^2)(1+y^2/x^2)-d^2y/x=0$$

$$\Rightarrow\quad (xy-c^2)(x^2+y^2)=d^2xy$$

To be Continued at Page 37

BITSAT
Practice Gallery

30 Important Unsolved Questions with Answer Key

1. The locus of the point z satisfying $\text{Re}\left(\frac{1}{z}\right) = k$, where k is a non-zero real number, is a/an
(a) straight line *(b)* circle
(c) ellipse *(d)* hyperbola

2. If $A = \begin{bmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\beta & \cos\beta\sin\beta \\ \cos\beta\sin\beta & \sin^2\beta \end{bmatrix}$ are two matrices such that the product AB is null matrix, then $\alpha - \beta$ is
(a) 0 *(b)* multiple of π
(c) an odd multiple of $\pi/2$ *(d)* None of these

3. The distance between the origin and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$, is
(a) $\frac{1}{\sqrt{5}}$ *(b)* $\frac{2}{\sqrt{5}}$ *(c)* $-\frac{1}{\sqrt{5}}$ *(d)* $\frac{2}{\sqrt{3}}$

4. The general solution of the equation $(\sqrt{3}-1)\sin\theta + (\sqrt{3}+1)\cos\theta = 2$ is
(a) $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{12}$ *(b)* $n\pi + (-1)^n \frac{\pi}{4} + \frac{\pi}{12}$
(c) $2n\pi \pm \frac{\pi}{4} - \frac{\pi}{12}$ *(d)* $n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}$

5. If the tangent at the point (x_1, y_1) to the parabola $y^2 = 4ax$ meets the parabola $y^2 = 4a(x + b)$ at Q and R, then the mid-point of QR is
(a) (x_1, y_1) *(b)* $(x_1 + b, y_1)$
(c) $(x_1 + b, y_1 + b)$ *(d)* $(x_1 - b, y_1 - b)$

6. Let $X = \{1, 2, 3, 4, 5\}$. Then, the number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is
(a) 5^2 *(b)* 3^5
(c) 2^5 *(d)* 5^3

7. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009}$ is equal to
(a) -2 *(b)* -1
(c) 1 *(d)* 2

8. A five digit number divisible by 3 is to be formed by using the digits 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done, is
(a) 216 *(b)* 240
(c) 600 *(d)* 3125

9. The middle term in the expansion of $\left(x^2 + \frac{1}{x^2} + 2\right)^n$ is
(a) $\frac{n}{[(n/2)!]^2}$ *(b)* $\frac{(2n)!}{[(n/2)!]^2}$
(c) $\frac{1\cdot 3\cdot 5 \ldots (2n+1)}{n!}\cdot 2^n$ *(d)* $\frac{(2n)!}{(n!)^2}$

10. If $\log\left(\frac{5c}{a}\right)$, $\log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in AP, where a, b, c are in GP, then a, b, c are the lengths of sides of a/an
(a) isosceles triangle *(b)* equilateral triangle
(c) scalene triangle *(d)* None of these

11. The sum of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots$ upto ∞ is equal to
(a) $2\log_e 2$ *(b)* $\log_e 2 - 1$
(c) $\log_e 2$ *(d)* $\log_e\left(\frac{4}{e}\right)$

12. If $a_1, a_2, a_3, \ldots, a_n$ are in GP, then $\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$ is equal to
(a) 2 *(b)* 4 *(c)* 0 *(d)* 1

13. Let $x_1, x_2, \ldots, x_n$ be n observations such that $\Sigma x_i^2 = 400$ and $\Sigma x_i = 80$. Then, the possible value of n is
(a) 12 *(b)* 9 *(c)* 18 *(d)* 15

14. Let $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then
(a) $P(B/A) = P(B) - P(A)$
(b) $P(A' - B') = P(A') - P(B')$
(c) $P(A \cup B') = P(A)'\,P(B)'$
(d) $P(A/B) = P(A) - P(B)$

15. If $x\cos\theta = y\cos\left(\theta + \frac{2\pi}{3}\right) = z\cos\left(\theta + \frac{4\pi}{3}\right)$, then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ is
(a) 1 *(b)* 0 *(c)* 2 *(d)* $3\cos\theta$

16. If p, q and r are simple propositions with truth values T, F and T then the truth value of $(\sim p \vee q) \wedge \sim r \Rightarrow p$ is
(a) false *(b)* true
(c) true, if r is false *(d)* true, if q is true

17. Let $f : R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$. Then, $\lim_{x \to 0} \left[\frac{f(1+x)}{f(1)}\right]^{1/x}$ equals

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

18. The value of $\lim_{x \to 0} \frac{\int_0^{x^2} \sin \sqrt{t}\, dt}{x^3}$ is

(a) 0 (b) 2/9 (c) 1/3 (d) 2/3

19. The area bounded between the parabolas $x^2 = \frac{y}{4}$, $x^2 = 9y$ and the straight line $y = 2$ is

(a) $20\sqrt{2}$ sq units (b) $\frac{10\sqrt{2}}{3}$ sq units

(c) $\frac{20\sqrt{2}}{3}$ sq units (d) $10\sqrt{2}$ sq units

20. The value of $\int_0^1 \frac{8 \log (1+x)}{1+x^2}\, dx$ is

(a) $\frac{\pi}{8} \log 2$ (b) $\frac{\pi}{2} \log 2$ (c) $-\log 2$ (d) $\pi \log 2$

21. Let $p(x)$ be a function defined on R such that $\lim_{x \to \infty} \frac{f(3x)}{f(x)} = 1$, $p'(x) = p'(1-x), \forall\, x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then, $\int_0^1 p(x)\, dx$ equals

(a) $\sqrt{41}$ (b) 21 (c) 41 (d) 42

22. The differential equation of the family of circles with fixed radius 5 units and centre lie on the line $y = 2$, is

(a) $(x-2)y'^2 = 25 - (y-2)^2$ (b) $(y-2)y'^2 = 25 - (y-2)^2$

(c) $(y-2)^2 y'^2 = 25 - (y-2)^2$ (d) $(x-2)^2 y'^2 = 25 - (y-2)^2$

23. The image of the line $\frac{x-1}{3} = \frac{y-3}{1} = \frac{z-4}{-5}$ in the plane $2x - y + z + 3 = 0$ is the line

(a) $\frac{x-3}{-3} = \frac{y+5}{-1} = \frac{z-2}{5}$ (b) $\frac{x+3}{-3} = \frac{y-5}{1} = \frac{z-2}{-5}$

(c) $\frac{x+3}{3} = \frac{y-5}{-1} = \frac{z+2}{5}$ (d) $\frac{x-3}{3} = \frac{y+5}{1} = \frac{z-2}{-5}$

24. If the planes $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ pass through a straight line, then the value of $a^2 + b^2 + c^2 + 2abc$ is

(a) 2 (b) 3

(c) 0 (d) 1

25. Let $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ be three vectors such that $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 4$ and angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{3}$, angle between $\mathbf{b}$ and $\mathbf{c}$ is $\frac{\pi}{3}$ and angle between $\mathbf{c}$ and $\mathbf{a}$ is $\frac{\pi}{3}$. The volume of the parallelopiped whose adjacent edges are represented by the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, is

(a) $24\sqrt{3}$ Cu units (b) $32\sqrt{3}$ Cu units

(c) $24\sqrt{2}$ Cu units (d) $32\sqrt{2}$ Cu units

26. From any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, if tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c \sin^2 \alpha + (g^2 + f^2) \cos^2 \alpha = 0$, then angle between the tangents is

(a) α (b) 2α (c) $\frac{\alpha}{2}$ (d) 3α

27. If $x + 2y = 3$ and $2x + y = 3$ touch the circle whose centre lies on a line $3x + 4y = 5$, then the possible centres of circle are

(a) $\left(\frac{5}{7}, \frac{5}{7}\right), \left(4, -\frac{3}{4}\right)$ (b) $\left(\frac{5}{3}, 0\right), (3, -1)$

(c) $\left(\frac{5}{3}, 0\right), \left(4, -\frac{3}{4}\right)$ (d) $\left(\frac{5}{7}, \frac{5}{7}\right), (3, -1)$

28. If the distance of a given point (α, β) from each of two straight lines through the origin is d, then the value of $(\alpha y - \beta x)^2$ is equal to

(a) $d^2(x^2 - y^2)$ (b) $d^2(x^2 + y^2)$

(c) $d^2(x + y)$ (d) $d^2(x - y)$

29. The point on the curve $3y = 6x - 5x^3$, normal at which passes through the origin, is

(a) $\left(1, \frac{1}{3}\right)$ (b) $\left(\frac{1}{3}, 1\right)$

(c) $\left(2, -\frac{28}{3}\right)$ (d) $\left(-1, \frac{1}{3}\right)$

30. The equation $16x^2 - 3y^2 - 32x + 12y - 44 = 0$ represents a hyperbola

(a) the length of whose transverse axis is $4\sqrt{3}$

(b) the length of whose conjugate axis is 4

(c) whose centre is $(-1, 2)$

(d) whose eccentricity is $\sqrt{\frac{19}{3}}$

Answers

1. (b)	2. (c)	3. (a)	4. (a)	5. (a)
6. (b)	7. (c)	8. (a)	9. (d)	10. (d)
11. (d)	12. (c)	13. (c)	14. (c)	15. (b)
16. (b)	17. (c)	18. (d)	19. (c)	20. (d)
21. (b)	22. (c)	23. (b)	24. (d)	25. (d)
26. (a)	27. (d)	28. (b)	29. (a)	30. (d)

Mind Shaker

A group of boys and girls are playing. 15 boys leave. There remain two girls for each boy. Then 45 girls leave. There remain five boys for each girl. How many boys were in the original group?

BRAiN TEASERS

To boost up your rank in JEE Main and Advanced

1. Find the sum of the series
$1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots$ to n terms.

2. Solve the following equations for x and y.
$4x^3+3x^2y+y^3=8,\ 2x^3-2x^2y+xy^2=1$

3. On a rod of length $a+b+c$, lengths a, b are measured at random. Find the probability that no point of the measured lines will coincide.

4. If the sides of a triangle are in AP, then prove that so also are the cotangents of half the angles.

5. Find the sum of the series
$\cos^3\alpha+\cos^3 2\alpha+\cos^3 3\alpha+\ldots$ to n terms.

6. Let the real numbers a, b, c, d satisfy the relations $a+b+c+d=6$ and $a^2+b^2+c^2+d^2=12$. Prove that $36\le 4(a^3+b^3+c^3+d^3)-(a^4+b^4+c^4+d^4)\le 48$.

7. Find the cube root of $72-32\sqrt{5}$.

8. If α and β are the roots of the equation $ax^2+bx+c=0$, then show that $\log(a-bx+cx^2)=\log a+(\alpha+\beta)x-\frac{\alpha^2+\beta^2}{2}x^2+\frac{\alpha^3+\beta^3}{3}x^3-\ldots$.

9. Find the sum of the squares and the cubes of the roots of the equation
$$x^3-px^2+qx-r=0.$$

10. Solve the following equations.
$$y^2+yz+z^2=49$$
$$z^2+zx+x^2=19$$
$$x^2+xy+y^2=39$$

Solutions

1. Let $S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots+\frac{3n-2}{5^{n-1}}$ ...(i)

Then, $\frac{1}{5}S=\frac{1}{5}+\frac{4}{5^2}+\frac{7}{5^3}+\frac{10}{5^4}+\ldots+\frac{3n-5}{5^{n-1}}+\frac{3n-2}{5^n}$ ...(ii)

On subtracting Eq. (ii) from Eq. (i), we get

$$\frac{4}{5}S=1+\left(\frac{3}{5}+\frac{3}{5^2}+\frac{3}{5^3}+\ldots+\frac{3}{5^{n-1}}\right)-\frac{3n-2}{5^n}$$

$$=1+\frac{3}{5}\left(1+\frac{1}{5}+\frac{1}{5^2}+\ldots+\frac{1}{5^{n-2}}\right)-\frac{3n-2}{5^n}$$

$$=1+\frac{3}{5}\cdot\frac{1-\frac{1}{5^{n-1}}}{1-\frac{1}{5}}-\frac{3n-2}{5^n}=1+\frac{3}{4}\left(1-\frac{1}{5^{n-1}}\right)-\frac{3n-2}{5^n}$$

$$=\frac{7}{4}-\frac{12n+7}{4\cdot 5^n}$$

$\therefore\quad S=\frac{35}{16}-\frac{12n+7}{16\cdot 5^{n-1}}$

2. Given equations are

$$4x^3+3x^2y+y^3=8 \quad \ldots(i)$$

and $$2x^3-2x^2y+xy^2=1 \quad \ldots(ii)$$

Put $y=mx$ in Eqs. (i) and (ii), we get

$$x^3(4+3m+m^3)=8 \quad \ldots(iii)$$

and $$x^3(2-2m+m^2)=1 \quad \ldots(iv)$$

On dividing Eq. (iii) by Eq. (iv), we get

$$\frac{4+3m+m^3}{2-2m+m^2}=8$$

$\Rightarrow\quad 4+3m+m^3=16-16m+8m^2$

$\Rightarrow\quad m^3-8m^2+19m-12=0$

$\Rightarrow\quad (m-1)(m-3)(m-4)=0$

$\Rightarrow\quad m=1, 3, 4$

When $m=1$, then from Eq. (iv),

$$x^3=1\ \Rightarrow\ x=1$$

$\therefore\quad y=1 \qquad [\because y = mx \text{ and } m = 1]$

When $m = 3$, then from Eq. (iv),

$$5x^3 = 1$$

$$\Rightarrow \quad x = \left(\frac{1}{5}\right)^{1/3}$$

$$\therefore \quad y = 3\left(\frac{1}{5}\right)^{1/3} \qquad [\because y = mx \text{ and } m = 3]$$

When $m = 4$, then from Eq. (iv),

$$10x^3 = 1 \Rightarrow x = \left(\frac{1}{10}\right)^{1/3}$$

$$\therefore \quad y = 4\left(\frac{1}{10}\right)^{1/3} \qquad [\because y = mx \text{ and } m = 4]$$

Hence, the complete solution is

$$x = 1, \left(\frac{1}{5}\right)^{1/3}, \left(\frac{1}{10}\right)^{1/3}$$

and $$y = 1, 3\left(\frac{1}{5}\right)^{1/3}, 4\left(\frac{1}{10}\right)^{1/3}$$

3. Let AB be the line and suppose $AP = x$ and $PQ = a$. Also, let a be measured from P towards B, so that x must be less than $b + c$. Again let $AP' = y$, $P'Q' = b$ and suppose $P'Q'$ measured from P' towards B', then y must be less than $a + c$.

Now, in favourable cases, we must have $AP' > AQ$ or else $AP > AQ'$.

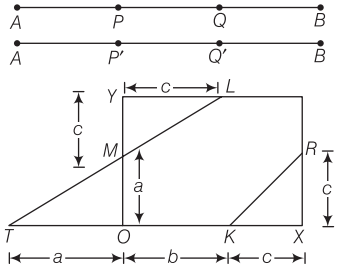
Hence, $$y > a + x \text{ or } x > b + y \qquad \ldots(i)$$

Again for all the possible cases, we must have

$$\left.\begin{aligned} x > 0 \text{ and } x < b + c \\ y > 0 \text{ and } y < a + b \end{aligned}\right\} \qquad \ldots(ii)$$

Take a pair of rectangular axes and make OX equal to $b + c$ and OY equal to $a + c$.

Draw the line $y = a + x$ represented by TML in the figure and the line $x = b + y$ represented by KR.



Then, YM, KX each equal to c and OM, OT equal to a.

Then, Eq. (i) is only satisfied by points in ΔMYL and ΔKXR, while Eq. (ii) is satisfied by any points within the rectangle OX, OY.

$$\therefore \text{ Required probability} = \frac{c^2}{(a + c)(b + c)}$$

4. Given that, $$a + c = 2b \qquad \ldots(i)$$

We have to prove that

$$\cot\frac{A}{2} + \cot\frac{C}{2} = 2\cot\frac{B}{2} \qquad \ldots(ii)$$

Now, Eq. (ii) is true, if

$$\sqrt{\frac{s(s-a)}{(s-b)(s-c)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = 2\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

On multiplying both sides by $\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$, we get

$$(s-a) + (s-c) = 2(s-b)$$

$$\Rightarrow \quad 2s - (a + c) = 2s - 2b$$

$$\Rightarrow \quad a + c = 2b$$

which is Eq. (i).

Hence, if Eq. (i) is true, then Eq. (ii) is also true.

5. We have, $$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$$

$$\Rightarrow \quad 4\cos^3\alpha = 3\cos\alpha + \cos 3\alpha$$

Similarly, $$4\cos^3 2\alpha = 3\cos 2\alpha + \cos 6\alpha$$

$$4\cos^3 3\alpha = 3\cos 3\alpha + \cos 9\alpha$$

$$\vdots \qquad \vdots \qquad \vdots$$

Let S be the given series, then

$$4S = (3\cos\alpha + \cos 3\alpha) + (3\cos 2\alpha + \cos 6\alpha) + (3\cos 3\alpha + \cos 9\alpha) + \ldots$$

$$= 3(\cos\alpha + \cos 2\alpha + \cos 3\alpha + \ldots) + (\cos 3\alpha + \cos 6\alpha + \cos 9\alpha + \ldots)$$

$$= \frac{3\cos\left\{\alpha + \left(\frac{n-1}{2}\right)\alpha\right\}\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}} + \frac{\cos\left\{3\alpha + \left(\frac{(n-1)}{2}\right)\cdot 3\alpha\right\}\sin\frac{n\cdot 3\alpha}{2}}{\sin\frac{3\alpha}{2}}$$

$$= \frac{3\cos\left(\frac{n+1}{2}\right)\alpha\sin\frac{n\alpha}{2}}{\sin\frac{\alpha}{2}} + \frac{\cos\left(\frac{3(n+1)}{2}\right)\alpha\sin\frac{3n\alpha}{2}}{\sin\frac{3\alpha}{2}}$$

6. We have, $4(a^3 + b^3 + c^3 + d^3) - (a^4 + b^4 + c^4 + d^4)$

$$= -[(a-1)^4 + (b-1)^4 + (c-1)^4 + (d-1)^4] + 6(a^2 + b^2 + c^2 + d^2) - 4(a + b + c + d) + 4$$

$$= -[(a-1)^4 + (b-1)^4 + (c-1)^4 + (d-1)^4] + 52$$

Now, introducing $x = a - 1$, $y = b - 1$, $z = c - 1$ and $t = d - 1$, we need to prove the inequalities

$16 \geq x^4 + y^4 + z^4 + t^4 \geq 4$, under the constraint

$$x^2 + y^2 + z^2 + t^2 = (a^2 + b^2 + c^2 + d^2) - 2(a + b + c + d) + 4 = 4 \qquad \ldots(i)$$

[we will not use in the value of $x + y + z + t$ though it can be found]

Now, the rightmost inequality in Eq. (i) follows from the power mean inequality.

i.e. $$x^4 + y^4 + z^4 + t^4 \geq \frac{(x^2 + y^2 + z^2 + t^2)^2}{4} = 4$$

For the other one, expanding the brackets, we note that

$$(x^2 + y^2 + z^2 + t^2)^2 = (x^4 + y^4 + z^4 + t^4) + q$$

where, q is a non-negative number.

$$\therefore \quad x^4 + y^4 + z^4 + t^4 \leq (x^2 + y^2 + z^2 + t^2)^2 = 16$$

Hence proved.

7. Let $(72-32\sqrt{5})^{1/3} = x - \sqrt{y}$ …(i)

Then, $(72+32\sqrt{5})^{1/3} = x + \sqrt{y}$ …(ii)

On multiplying Eqs. (i) and (ii), we get

$(5184-1024\times 5)^{1/3} = x^2 - y$

$\Rightarrow \quad 4 = x^2 - y$ …(iii)

Again, $72-32\sqrt{5} = x^3 - 3x^2\sqrt{y} + 3xy - y\sqrt{y}$

$\therefore \quad x^3 + 3xy = 72$ …(iv)

From Eqs. (iii) and (iv),

$72 = x^3 + 3x(x^2 - 4)$

$\Rightarrow \quad x^3 - 3x = 18$

$\therefore \quad x = 3$ and $y = 5$

Hence, $(72-32\sqrt{5})^{1/3} = 3 - \sqrt{5}$

8. Since, $\alpha+\beta = \dfrac{-b}{a}$ and $\alpha\beta = \dfrac{c}{a}$

$\therefore \; a - bx + cx^2 = a\left\{1 - \dfrac{b}{a}x + \dfrac{c}{a}x^2\right\}$

$= a\{1 + (\alpha+\beta)x + \alpha\beta x^2\}$

$= a(1+\alpha x)(1+\beta x)$

$\therefore \log(a - bx - cx^2) = \log a + \log(1+\alpha x) + \log(1+\beta x)$

$= \log a + \alpha x - \dfrac{\alpha^2 x^2}{2} + \dfrac{\alpha^3 x^3}{3} - \ldots + \beta x - \dfrac{\beta^2 x^2}{2} + \dfrac{\beta^3 x^3}{3} - \ldots$

$= \log a + (\alpha+\beta)x - \dfrac{\alpha^2+\beta^2}{2}x^2 + \dfrac{\alpha^3+\beta^3}{3}x^3 - \ldots$

9. Let the roots of the equation be a, b and c.

Then, $a+b+c = p$ and $ab+bc+ca = q$

Now, $a^2+b^2+c^2 = (a+b+c)^2 - 2(ab+bc+ca) = p^2 - 2q$

Again, substitute a, b, c for x in the given equation and add them, we get

$a^3+b^3+c^3 - p(a^2+b^2+c^2) + q(a+b+c) - 3r = 0$

$\Rightarrow a^3+b^3+c^3 = p(p^2-2q) - pq + 3r = p^3 - 3pq + 3r$

10. Given equations are

$y^2 + yz + z^2 = 49$ …(i)

$z^2 + zx + x^2 = 19$ …(ii)

and $x^2 + xy + y^2 = 39$ …(iii)

On subtracting Eq. (ii) from Eq. (i), we get

$y^2 - x^2 + z(y-x) = 30$

$\Rightarrow \quad (y-x)(x+y+z) = 30$ …(iv)

Similarly, from Eqs. (i) and (iii), we get

$(z-x)(x+y+z) = 10$ …(v)

On dividing Eq. (iv) by Eq. (v), we get

$$\frac{y-x}{z-x} = 3 \Rightarrow y = 3z - 2x$$

On putting the value of y in Eq. (iii), we get

$x^2 - 3xz + 3z^2 = 13$ …(vi)

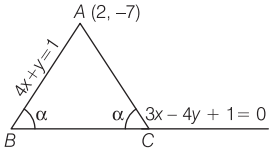
On solving Eqs. (ii) and (vi), we get

$x = \pm 2,\ y = \pm 5,\ z = \pm 3$

and $x = \pm\dfrac{11}{\sqrt{7}},\ y = \mp\dfrac{19}{\sqrt{17}},\ z = \pm\dfrac{1}{\sqrt{7}}$

Continued from Page 32

29. ***(d)*** We have, $\tan\alpha = \left|\dfrac{m-m'}{1+mm'}\right|$, where m and m' are the slopes of $3x - 4y + 1 = 0$ and $4x + y = 1$, respectively.



$\therefore \quad m = \dfrac{3}{4}$ and $m' = -4$

Also, $\tan\alpha = \left|\dfrac{\frac{3}{4}+4}{1-3}\right| = \dfrac{19}{8}$

Now, the equation of AC is either

$y + 7 = \tan(\theta - \alpha)(x - 2)$ …(i)

or $y + 7 = \tan(\theta + \alpha)(x - 2)$ …(ii)

where, $\tan\theta = m = \dfrac{3}{4}$

Now, $\tan(\theta - \alpha) = \dfrac{\frac{3}{4} - \frac{19}{8}}{1 + \frac{57}{32}} = -\dfrac{52}{89}$

and $\tan(\theta + \alpha) = \dfrac{\frac{3}{4} + \frac{19}{8}}{1 - \frac{57}{32}} = -4$

which is the same as slope of AB.

$\therefore$ Equation of the required line is

$y + 7 = -\dfrac{52}{89}(x - 2)$

$\Rightarrow \quad 52x + 89y + 519 = 0$

30. ***(b)***

p	q	$\sim p$	$\sim p \wedge q$	$q \to p$	$\sim(q \to p)$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	F	T	F

Hence, $(p, q) = (F, T)$

TARGET JEE 2017

TEST RIDER

Comprehensive Simulator Test Series for JEE Main & Advanced

JEE ADVANCED

(PAPER ONE PREP UP)

Mock Questions from Complete Syllabus with Complete Solutions

FULL TEST 1

(COMPLETE SYLLABUS) FULLY SOLVED

PAPER I

SECTION 1 (MM : 18)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. If the line $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$ is the hypotenuse of an isosceles right angled triangle whose opposite vertex is (7, 2, 4). Then, which of the following is not the side of the triangle?

(a) $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$

(b) $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$

(c) $\frac{x-7}{3}=\frac{y-2}{5}=\frac{z-4}{-1}$

(d) None of the above

2. The product of all values of x satisfying the equation

$$\sin^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)=\cot\left\{\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right\}+\frac{\pi}{2}\text{ is}$$

(a) 9 *(b)* −9 *(c)* −3 *(d)* −1

3. The distance between the two parallel lines is l unit and a point A is chosen to lie between the lines at a distance d from one of them. If ΔABC is equilateral with B on one line and C on the other parallel line, then the length of the side of the equilateral triangle is

(a) $\frac{2}{3}\sqrt{d^2+d+1}$ *(b)* $2\sqrt{\frac{d^2-d+1}{3}}$

(c) $2\sqrt{d^2-d+1}$ *(d)* $\sqrt{d^2-d+1}$

4. Let P_i and P_i' be the feet of the perpendiculars drawn from foci S and S' on a tangent T_i to an ellipse whose length of semi-major axis is 20. If $\sum_{i=1}^{10}(SP_i)(S'P_i')=2560$, then its eccentricity is

(a) $\frac{1}{5}$ *(b)* $\frac{2}{5}$

(c) $\frac{3}{5}$ *(d)* $\frac{4}{5}$

5. Let C_1 and C_2 be concentric circles of radii 1 and 8/3 respectively, having centre at (3, 0) on the argand plane. If the complex number z satisfies the inequality $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right)>1$, then

(a) z lies outside C_1 but inside C_2
(b) z lies inside of both C_1 and C_2
(c) z lies outside of both C_1 and C_2
(d) None of the above

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks	:	+1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks	:	0 If none of the bubbles is darkened.
Negative Marks	:	−2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

6. Let the straight line passing through $T(\alpha, 2)$ meets the ellipse $4x^2 + 9y^2 = 36$ at A and D and meets the coordinate axes at B and C such that TA, TB, TC and TD (take in that order) are in GP, then the possible value of α can be
(a) -7 *(b)* 5 *(c)* 8 *(d)* 10

7. Let $f(x) = [x]^2 + [x+1] - 3$, where $[x] \le x$, then
(a) $f(x)$ is many-one and into function
(b) $f(x) = 0$ for infinite number of values of x
(c) $f(x) = 0$ for only two real values
(d) None of the above

8. If the function $f(x) = 3 + 2(a+1)\,x + (a^2+1)\,x^2 - x^3$ has a local minimum at $x = x_1$ and local maximum at $x = x_2$ such that $x_1 < 2 < x_2$, then a belongs to the intervals
(a) $\left(-\infty, \frac{-3}{2}\right)$ *(b)* $\left(\frac{-3}{2}, 1\right)$ *(c)* $(0, \infty)$ *(d)* $(1, \infty)$

9. The value(s) of x satisfying the equation $\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$ is /are given by (n is any integer)
(a) $n\pi - 1$ *(b)* $n\pi$ *(c)* $n\pi + 1$ *(d)* $2n\pi + 1$

10. If $A(z_1), B(z_2), C(z_3)$ and $D(z_4)$ lies on $|z| = 4$ (taken in order), where $z_1 + z_2 + z_3 + z_4 = 0$, then
(a) maximum area of quadrilateral, $ABCD = 32$
(b) the ΔABC is right angled
(c) the quadrilateral $ABCD$ is rectangle
(d) maximum area of quadrilateral $ABCD = 16$

11. In a question, a student was given to find the derivative of the product of two functions f and g. The student by mistake thought $(fg)' = f'g'$ for his question $f(x) = x^3$ and he got the correct answer. If $g(4) = 1$, then which of the following is / are correct possibilities (where, $g(x) > 0$)?
(a) $g(x)$ is decreasing in $(-\infty, 3)$
(b) $g(x)$ is increasing in $(3, \infty)$
(c) $g(x)$ is decreasing in $(3, \infty)$
(d) $g(x)$ is increasing in $(-\infty, 3)$

12. A square is inscribed in a circle, if P_1 is the probability that a randomly chosen point of the circle lies within the square and P_2 is the probability that the point lies outside the square, then
(a) $P_1 < P_2$ *(b)* $P_1 = P_2$ *(c)* $P_1 > P_2$ *(d)* $P_1^2 - P_2^2 < \frac{1}{3}$

13. If $f(x+y) = f(x) \cdot f(y)$ for all x, y and $f(1) = 2$ and $a_r = f(r)$, for $r \in N$, then the coordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4, may be
(a) (a_1, a_2) *(b)* $(a_1, -a_2)$
(c) (a_1, a_1) *(d)* None of these

SECTION 3 (MM : 12)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+ 3 If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0 In all other cases.

14. If the function $f(x) = x^4 + bx^2 + 8x + 1$ has a horizontal tangent and a point of inflection for the same value of x, then the value of $|b|$ is equal to .... .

15. If $k \sin 17^\circ \sin 43^\circ \sin 77^\circ = \cos 39^\circ$, then the value of k is .... .

16. The number of ordered triplet solutions (x, y, z) of the system $x + y + z = 4$, $x^2 + y^2 + z^2 = 14$ and $x^3 + y^3 + z^3 = 34$ is/are .... .

17. Let $A = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \ldots + \frac{1}{\sqrt{10000}}$, then the value of $\left[\frac{[A]}{50}\right]$ is ...., where $[\cdot]$ is greatest integer function.

18. Let $I = \int_2^3 \frac{dx}{\sqrt{x^3 - 3x^2 + 5}}$, then the value of $[I + \sqrt{3}]$, where $[\cdot]$ represents greatest integral function, is ... .

PAPER II

SECTION 1 (MM : 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are any three non-coplanar vectors, then the equation $[\mathbf{b}\times\mathbf{c}\ \ \mathbf{c}\times\mathbf{a}\ \ \mathbf{a}\times\mathbf{b}]\,x^2 + [\mathbf{a}+\mathbf{b}\ \ \mathbf{b}+\mathbf{c}\ \ \mathbf{c}+\mathbf{a}]\,x + 1 + [\mathbf{b}-\mathbf{c}\ \ \mathbf{c}-\mathbf{a}\ \ \mathbf{a}-\mathbf{b}] = 0$ has
(a) real and distinct roots *(b)* real roots
(c) equal roots *(d)* imaginary roots

2. In ΔABC, $\angle ABC = 120°$, $AB = 3$ and $BC = 4$. If perpendiculars constructed to the side AB at A and to the side BC at C meets at D, then CD is equal to
(a) 3 *(b)* $\frac{8\sqrt{3}}{3}$ *(c)* 5 *(d)* $\frac{10\sqrt{3}}{3}$

3. If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in GP with same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3)
(a) lie on a straight line *(b)* lie on an ellipse
(c) lie on a circle *(d)* are vertices of a triangle

4. If $\prod_{k=1}^{n} \cos\frac{x}{2^k} = \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$ and

$$f(x) = \begin{cases} \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{1}{2^k} \tan\left(\frac{x}{2^k}\right), & x \in (0,\pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, & x = \frac{\pi}{2} \end{cases}$$

Then, which one of the following is true?
(a) $f(x)$ has non-removable discontinuity of finite type at $x = \frac{\pi}{2}$
(b) $f(x)$ has removable discontinuity at $x = \frac{\pi}{2}$
(c) $f(x)$ is continuous at $x = \frac{\pi}{2}$
(d) $f(x)$ has non-removable discontinuity of infinite type at $x = \frac{\pi}{2}$

5. The three sides of a trapezium, other than base, are equal, each being 8 cm. The area of the trapezium, when it is maximum, is
(a) $24\sqrt{3}$ cm^2
(b) $48\sqrt{3}$ cm^2
(c) $72\sqrt{2}$ cm^2
(d) None of the above

6. If r, s, t are prime numbers and p, q are the positive integers such that the LCM of p, q is $r^2t^4s^2$, then the number of ordered pair (p, q) is
(a) 252 *(b)* 254
(c) 225 *(d)* 224

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

7. If $(\sin^{-1} x + \sin^{-1} w)(\sin^{-1} y + \sin^{-1} z) = \pi^2$, then $D = \begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix}$ (where, $N_1, N_2, N_3, N_4 \in N$)

(a) has maximum value of 2 *(b)* has minimum value of 0
(c) 16 different D are possible *(d)* has minimum value of -2

8. If $f(2 - x) = f(2 + x)$ and $f(4 - x) = f(4 + x)$ for all x and $f(x)$ is a function for which $\int_0^2 f(x)\,dx = 5$, then $\int_0^{50} f(x)\,dx$ is equal to

(a) 125 *(b)* $\int_{-4}^{46} f(x)\,dx$ *(c)* $\int_1^{51} f(x)\,dx$ *(d)* $\int_2^{52} f(x)\,dx$

9. If $f(x) = \begin{cases} x + a; & x \geq 0 \\ 2 - x; & x < 0 \end{cases}$, $g(x) = \begin{cases} \{x\}; & x < 0 \\ \sin x + b; & x \geq 0 \end{cases}$ and $f(g(x))$ is continuous at $x = 0$, then which of the following is /are true (where, $\{x\}$ represents the fractional part function)?

(a) If $b = 1$, then a can take any real value
(b) If $b < -1$, then $a + b = 1$
(c) No values of a and b are possible
(d) There exist finite ordered pairs (a, b)

10. If the equation whose roots are squares of the roots of the cubic equation $x^3 - ax^2 + bx - 1 = 0$, is identical with the given cubic equation, then

(a) $a = 0, b = 3$
(b) $a = b = 0$
(c) $a = b = 3$
(d) a, b are roots of $x^2 + x + 2 = 0$

11. If point M moved on the circle $(x - 4)^2 + (y - 8)^2 = 20$, then, it broke away from it and moving along a tangent to the circle, cuts the X-axis at the point $(-2, 0)$. The coordinates of a point on the circle at which the moving point broke away, is

(a) $\left(\frac{42}{5}, \frac{36}{5}\right)$ *(b)* $\left(-\frac{2}{5}, \frac{44}{5}\right)$ *(c)* $(6, 4)$ *(d)* $(2, 4)$

12. If P is a point on a hyperbola, then

(a) locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'$ (S and S' are foci) is tangent at vertex
(b) locus of excentre of the circle described opposite to $\angle S'$ is hyperbola
(c) locus of excentre of the circle described opposite to $\angle P$ for $\Delta PSS'$ (S, S' are foci) is hyperbola
(d) locus of excentre of the circle described opposite to $\angle S'$, is tangent at vertex

13. Let P_1 denotes the equation of a plane to which the vector $\hat{\mathbf{i}} + \hat{\mathbf{j}}$ is normal and which contains the line whose equation is $\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$ and P_2 denotes the equation of the plane containing the line L and a point with position vector $\hat{\mathbf{j}}$. Which of the following holds good?

(a) The equation of P_1 is $x + y = 2$
(b) The equation of P_2 is $\mathbf{r} \cdot (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}) = 2$
(c) The acute angle between P_1 and P_2 is $\cot^{-1}(\sqrt{3})$
(d) The angle between the plane P_2 and the line L is $\tan^{-1}(\sqrt{3})$

14. Let $f : (0, \infty) \to R$ be given by $f(x) = \int_{1/x}^{x} e^{-\left(t + \frac{1}{t}\right)} \frac{dt}{t}$, then

(a) $f(x)$ is monotonically increasing on $[1, \infty)$
(b) $f(x)$ is monotonically decreasing on $[0, 1)$
(c) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$
(d) $f(2^x)$ is an odd function of x on R

SECTION 3 (MM : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+ 3 If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0 In all other cases.

Passage 1 (Q. Nos. 15-16) In a class of 10 students, probability of exactly i students, passing an examination is directly proportional to i^2.

15. If a student is selected at random, then the probability that he has passed the examination, is

(a) $\frac{1}{7}$ *(b)* $\frac{11}{35}$
(c) $\frac{11}{14}$ *(d)* None of these

16. If a student selected at random is found to have passed the examination, then the probability that he was the only student who has passed the examination is

(a) $\frac{1}{3025}$ *(b)* $\frac{1}{605}$
(c) $\frac{1}{275}$ *(d)* $\frac{1}{121}$

Passage 2 (Q. Nos. 17-18) Consider the function

$$f(x)=\begin{vmatrix} a^2+x & ab & ac \\ ab & b^2+x & bc \\ ac & bc & c^2+x \end{vmatrix}$$

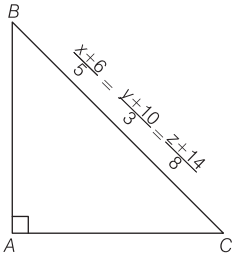
17. Which of the following is true?
(a) $f(x)=0$ and $f'(x)=0$ have one positive common root
(b) $f(x)=0$ and $f'(x)=0$ have one negative common root
(c) $f(x)=0$ and $f'(x)=0$ have no common root
(d) None of the above

18. Which of the following is true?
(a) $f(x)$ has one +ve point of maxima
(b) $f(x)$ has one –ve point of minima
(c) $f(x)=0$ has three distinct roots
(d) Local minimum value of $f(x)$ is zero

Answers with Explanation

Paper I

1. ***(c)*** Let ABC be the given isosceles right angled triangle with vertex $A(7, 2, 4)$ and hypotenuse BC, whose equation is $\frac{x+6}{5}=\frac{y+10}{3}=\frac{z+14}{8}$.



Clearly, the coordinates of B is given by
$(5\lambda-6, 3\lambda-10, 8\lambda-14)$, for some $\lambda\in R$
Now DR's of line AB are $(5\lambda-13, 3\lambda-12, 8\lambda-18)$
Since, DR's of line BC are 5, 3, 8 and angle between AB and BC is $\frac{\pi}{4}$.

$$\therefore \cos\frac{\pi}{4}=\frac{(5\lambda-13)\cdot 5+(3\lambda-12)\cdot 3+(8\lambda-18)\cdot 8}{\sqrt{5^2+3^2+8^2}\sqrt{(5\lambda-13)^2+(3\lambda-12)^2+(8\lambda-18)^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}}=\frac{98\lambda-245}{\sqrt{98}\sqrt{98\lambda^2-490\lambda+637}}$$

On squaring both sides, we get

$$\frac{98(98\lambda^2-490\lambda+637)}{2}=(98\lambda-245)^2$$
$$\Rightarrow 49(98\lambda^2-490\lambda+637)=(49)^2(2\lambda-5)^2$$
$$\Rightarrow (49)^2(2\lambda^2-10\lambda+13)=(49)^2(2\lambda-5)^2$$
$$\Rightarrow 2\lambda^2-10\lambda+13=4\lambda^2+25-20\lambda$$
$$\Rightarrow 2\lambda^2-10\lambda+12=0$$
$$\Rightarrow \lambda^2-5\lambda+6=0$$
$$\Rightarrow \lambda=3, 2$$

Hence, equation of lines are $\frac{x-7}{2}=\frac{y-2}{-3}=\frac{z-4}{6}$
and $\frac{x-7}{3}=\frac{y-2}{6}=\frac{z-4}{2}$.

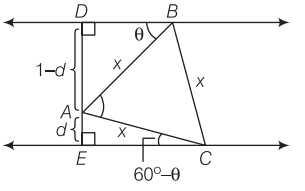
2. ***(a)*** Given equation can be written as

$$\frac{\pi}{2}-\cos^{-1}\cos\left(\frac{2x^2+10|x|+4}{x^2+5|x|+3}\right)=\cot\left\{\cot^{-1}\left(\frac{2-18|x|}{9|x|}\right)\right\}+\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2}-\frac{2(x^2+5|x|+3)-2}{x^2+5|x|+3}=\frac{2-18|x|}{9|x|}+\frac{\pi}{2}$$

$$\Rightarrow \frac{\pi}{2}-2+\frac{2}{x^2+5|x|+3}=\frac{2}{9|x|}-2+\frac{\pi}{2}$$
$$\Rightarrow x^2+5|x|+3=9|x|$$
$$\Rightarrow x^2-4|x|+3=0$$
$$\Rightarrow |x|=1, 3$$
$$\Rightarrow x=\pm 1, \pm 3$$

3. ***(b)*** Let x be the length of side of the equilateral triangle.
Clearly, $\angle ACE=180^\circ-(\theta+60^\circ+60^\circ)$



$$=180^\circ-\theta-120^\circ=60^\circ-\theta$$

Now, in ΔAEC, we have

$$\sin(60^\circ-\theta)=\frac{d}{x} \qquad \ldots(i)$$

and in ΔADB, we have

$$\sin\theta=\frac{1-d}{x} \qquad \ldots(ii)$$

From Eq. (i), we get

$$\frac{\sqrt{3}}{2}\cos\theta-\frac{1}{2}\sin\theta=\frac{d}{x}$$
$$\Rightarrow \frac{\sqrt{3}}{2}\sqrt{x^2-(1-d)^2}-\frac{1}{2}(1-d)=d \qquad \text{[using Eq. (ii)]}$$
$$\Rightarrow \sqrt{x^2-(1-d)^2}=\frac{(1+d)}{\sqrt{3}}$$

On squaring both sides, we get

$$x^2-(1-d)^2=\frac{(1+d)^2}{3}$$
$$\Rightarrow x^2=\frac{1}{3}(4d^2-4d+4)=\frac{4(d^2-d+1)}{3}$$
$$\Rightarrow x=2\sqrt{\frac{d^2-d+1}{3}}$$

4. ***(c)*** We know that, the product of perpendiculars from foci upon any tangent of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is b^2

$$\therefore SP_i\, S'P_i'=b^2$$

Now, $\sum_{i=1}^{10}(SP_i)(S'P_i')=2560 \Rightarrow \sum_{i=1}^{10}b^2=2560$

$$\Rightarrow 10\times b^2=2560 \Rightarrow b^2=256 \Rightarrow b=16$$

$\because \quad e^2 = 1 - \frac{b^2}{a^2}$

$\therefore \quad e^2 = 1 - \frac{(16)^2}{(20)^2} = \frac{400 - 256}{400} = \frac{144}{400}$

$\Rightarrow \quad e = \frac{12}{20} = \frac{3}{5}$

5. ***(a)*** We have, $\log_{1/3}\left(\frac{|z-3|^2+2}{11|z-3|-2}\right) > 1$

$\Rightarrow \quad \frac{|z-3|^2+2}{11|z-3|-2} < \frac{1}{3}$

$\Rightarrow \quad 3|z-3|^2 + 6 < 11|z-3| - 2$

$\Rightarrow \quad 3x^2 - 11x + 8 < 0$, where $x = |z-3|$

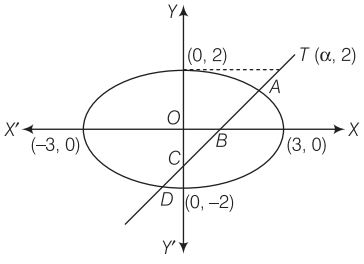
$\Rightarrow \quad (3x-8)(x-1) < 0$

$\Rightarrow \quad 1 < x < \frac{8}{3}$

$\Rightarrow \quad 1 < |z-3| < \frac{8}{3}$

Hence, z lies between the two concentric circles.

6. ***(a, c, d)*** We have, $4x^2 + 9x^2 = 36 \Rightarrow \frac{x^2}{9} + \frac{y^2}{4} = 1$



Let inclination of line be 'θ'.

Let $TA = r_1, TB = r_2, TC = r_3$ and $TD = r_4$

A and D are $(\alpha + r\cos\theta, 2 + r\sin\theta)$ with r_1 and r_4 satisfy ellipse $4x^2 + 9y^2 = 36$

$r^2(4\cos^2\theta + 9\sin^2\theta) + r(8\alpha\cos\theta + 36\sin\theta) + 4\alpha^2 = 0$

have roots r_1 and r_4

$$r_1 r_4 = \frac{4\alpha^2}{4\cos^2\theta + 9\sin^2\theta} = \frac{4\alpha^2}{4 + 5\sin^2\theta}$$

$$B = (\alpha + r_2\cos\theta, 2 + r_2\sin\theta) = (x_0, 0)$$

$$C = (\alpha + r_3\cos\theta, 2 + r_3\sin\theta) = (0, y_0)$$

Hence, $r_2 = \frac{-\alpha}{\cos\theta}, r_3 = \frac{-2}{\sin\theta}$

r_1, r_2, r_3, r_4 are GP $\Rightarrow r_1 r_4 = r_2 r_3$

$\Rightarrow \quad 2\alpha = 4\cot\theta + 9\tan\theta \Rightarrow |\alpha| \geq 6$

$\therefore \quad \alpha = -7, 8, 10$

7. ***(a, b)*** We have, $f(x) = [x]^2 + [x+1] - 3$

$\Rightarrow \quad f(x) = [x]^2 + [x] + 1 - 3$

$\Rightarrow \quad f(x) = ([x] + 2)([x] - 1)$

$\Rightarrow \quad f(x) = 0$, then $[x] = -2$

and $\quad [x] = 1$

$\because \quad -2 \leq x < -1$ and $1 \leq x < 2$

Hence, $f(x)$ is many one and into and $f(x) = 0$ for infinite values of x.

8. ***(a, d)*** We have, $f(x) = 3 + 2(a+1)x + (a^2+1)x^2 - x^3$

$\Rightarrow \quad f'(x) = 2(a+1) + 2(a^2+1)x - 3x^2$

Now, 2 lies between x_1 and x_2.

$\therefore \quad f(2) > 0$

$2(a+1) + 2(a^2+1)\,2 - 3(2)^2 > 0$

$\Rightarrow \quad 2a^2 + a - 3 > 0$

$\Rightarrow \quad 2a^2 + 3a - 2a - 3 > 0$

$\Rightarrow \quad (2a+3)(a-1) > 0$

$\Rightarrow \quad a < \frac{-3}{2}$ or $a > 1$

$\therefore \quad a \in \left(-\infty, \frac{-3}{2}\right) \cup (1, \infty)$

9. ***(a, b, c)*** We have,

$$\sin^{-1}|\sin x| = \sqrt{\sin^{-1}|\sin x|}$$

The solution of $y = \sqrt{y}$ is $y = 0$ or $y = 1$

$\therefore \quad \sin^{-1}|\sin x| = 0$ or $\sin^{-1}|\sin x| = 1$

If $\quad \sin^{-1}|\sin x| = 0, \; x = n\pi$

And if $\sin^{-1}|\sin x| = 1$

$\Rightarrow \quad x = 1$ or $\pi - 1$

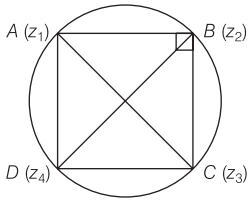
[In the interval $(0, \pi)$]

But $\sin^{-1}|\sin x|$ is periodic with period π, so

$$x = n\pi + 1 \text{ or } n\pi - 1$$

10. ***(a, b, c)*** We have,

$$z_1 + z_2 + z_3 + z_4 = 0$$



$\Rightarrow \quad z_1 + z_3 = -z_2 - z_4$

$\Rightarrow \quad \frac{z_1 + z_3}{2} = \frac{-z_2 - z_4}{2}$

Quadrilateral with vertices $z_1, z_3, -z_2, -z_4$ is a parallelogram but it is inscribed in circle, so it must be rectangle.

If $ABCD$ is a rectangle, then, ΔABC is right angled.

$\therefore$ Maximum area of quadrilateral $ABCD$

$$= \frac{1}{2}(2r)^2 = 2r^2 = 2 \times 16 = 32 \qquad [\because r = 4]$$

11. ***(c, d)*** We have, $(fg)' = f'g'$

$\Rightarrow \quad f'(x)g(x) + f(x)g'(x) = f'(x)g'(x)$

$\Rightarrow \quad 3x^2 g(x) + x^3 g'(x) = 3x^2 g'(x) \qquad [\because f(x) = x^3]$

$\Rightarrow \quad g'(x) = \frac{-3}{(x-3)}g(x)$

Integrating both side, we get

$$\log g(x) = -3\log(x-3) + c$$

Given, $g(4) = 1, c = 0$

$$\log g(x) = -3\log(x-3)$$

$$g(x) = \frac{1}{(x-3)^3}$$

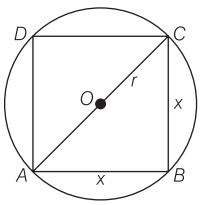
$$g(x) = \begin{cases} \frac{1}{(x-3)^3}, & x > 3 \\ \frac{-1}{(x-3)^3}, & x < 3 \end{cases}$$

$$g'(x) = \begin{cases} \dfrac{-3}{(x-3)^4}, & x > 3 \\ \dfrac{3}{(x-3)^4}, & x < 3 \end{cases}$$

Hence, $g(x)$ is decreasing in $x \in (3, \infty)$ and $g(x)$ is increasing in $x \in (-\infty, 3)$.

12. ***(c, d)*** If r is the radius of the circle and if x is the side of the square, then

$$x^2 + x^2 = (2r)^2$$
$$\Rightarrow \quad x^2 = 2r^2$$



Now, $P_1 = \dfrac{\text{Area of square}}{\text{Area of circle}} = \dfrac{x^2}{\pi r^2} = \dfrac{2r^2}{\pi r^2} = \dfrac{2}{\pi}$

$\therefore \quad P_2 = 1 - P_1 = 1 - \dfrac{2}{\pi} = \dfrac{\pi - 2}{\pi}$

$\therefore \quad 2 > \pi - 2$

$\Rightarrow \quad \dfrac{2}{\pi} > \dfrac{\pi - 2}{\pi} \Rightarrow P_1 > P_2$

Also, $P_1^2 - P_2^2 = (P_1 + P_2)(P_1 - P_2) = \dfrac{4 - \pi}{\pi}$

$$= \frac{4}{\pi} - 1 < \frac{1}{3}$$

Hence, $P_1^2 - P_2^2 < \dfrac{1}{3}$

13. ***(a, b)*** Given parabola is $y^2 = 8x$. ...(i)

Here, $a = 2$, let $P(2t^2, 4t)$ be a point on parabola (i) and S be the focus.

Given, $SP = 4$

$\therefore \quad a(1 + t^2) = 4$

$2(1 + t^2) = 4 \Rightarrow t = \pm 1$

$P \equiv (2, 4)$ or $(2, -4)$

Given, $f(x + y) = f(x)\, f(y)$ for all x and y ...(ii)

Given, $f(1) = 2$

From Eq. (ii)

$f(2), f(1)\, f(1) = (2)^2 = 4$

Similarly, $f(x) = 2^n$

$\because \quad a_r = f(r)$

$\therefore \quad a_1 = f(1) = 2$

$a_2 = f(2) = 4$

Hence, $P \equiv (a_1, a_2)$ or $(a_1, -a_2)$

14. ***(6)*** We have, $f(x) = x^4 + bx^2 + 8x + 1$

$$f'(x) = 4x^3 + 2bx + 8$$
$$f''(x) = 12x^2 + 2b$$

For point of inflection,

$$f''(x) = 0 \Rightarrow 12x^2 + 2b = 0$$
$$\Rightarrow \quad b = -6x^2$$

For horizontal tangent,

$$f'(x) = 0$$
$$\Rightarrow \quad 4x^3 + 2bx + 8 = 0 \Rightarrow 4x^3 + 2(-6x^2)x + 8 = 0$$
$$\Rightarrow \quad 4x^3 - 12x^3 + 8 = 0$$
$$\Rightarrow \quad x^3 = 1 \Rightarrow x = 1$$
$$\therefore \quad b = -6x^2 \Rightarrow b = -6 \text{ and } |b| = 6$$

15. ***(4)*** Now, $\sin 17^\circ \sin 43^\circ \sin 77^\circ$

$$= \sin 17^\circ \sin(60^\circ - 17^\circ)\sin(60^\circ + 17^\circ)$$
$$= \frac{1}{4}\sin 51^\circ = \frac{1}{4}\cos 39^\circ$$

$\Rightarrow 4\sin 17^\circ \sin 43^\circ \sin 77^\circ = \cos 39^\circ$

$\therefore \quad k = 4$

16. ***(6)*** Let function $f(t) = t^3 + at^2 + bt + c = 0$, has roots x, y, z

Then, $x + y + z = -a \Rightarrow a = -4$

Also, $x^2 + y^2 + z^2 = (x + y + z)^2 - 2(xy + yz + zx)$

$$\Rightarrow \quad 14 = (-4)^2 - 2(xy + yz + zx)$$
$$\Rightarrow \quad xy + yz + zx = 1 = b$$

And $x^3 + y^3 + z^3 - 4(x^2 + y^2 + z^2)$

$$+ (x + y + z) + 3c = 0$$

$\Rightarrow 34 - 4(14) + 4 + 3c = 0 \Rightarrow c = 6$

$\therefore f(t) = t^3 - 4t^2 + t + 6 = (t + 1)(t - 2)(t - 3)$

$\therefore$ Number of order triplets are 6.

17. ***(3)*** $\dfrac{1}{\sqrt{k}} = \dfrac{2}{\sqrt{k} + \sqrt{k}} < \dfrac{2}{\sqrt{k} + \sqrt{k-1}} = 2\sqrt{k} - \sqrt{k-1}$

Putting $k = 2, 3, 4, \ldots 10000$ and adding, we get

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \ldots + \frac{1}{\sqrt{10000}} < 2(\sqrt{10000} - \sqrt{1})$$
$$\Rightarrow \quad A < 2(100 - 1)$$
$$\Rightarrow \quad A < 198$$

Similarly, $\dfrac{1}{\sqrt{k}} = \dfrac{2}{\sqrt{k} + \sqrt{k}} > \dfrac{2}{\sqrt{k+1} + \sqrt{k}}$

$$= 2(\sqrt{k+1} - \sqrt{k})$$
$$\Rightarrow \quad A > 2(\sqrt{10001} - \sqrt{2}) > 197$$
$$\therefore \quad 197 < A < 198 \Rightarrow [A] = 197$$

Now, $\left[\dfrac{[A]}{50}\right] = \left[\dfrac{197}{50}\right] = 3$

18. ***(2)*** Let $f(x) = x^3 - 3x^2 + 5$

$f'(x) = 3x^2 - 6x$

$f'(x) = 3x(x - 2) > 0$, as $x \in [2, 3]$

$$\Rightarrow \quad \int_2^3 \frac{1}{\sqrt{5}}dx < \int_2^3 \frac{dx}{\sqrt{x^3 - 3x^2 + 5}} < \int_2^3 dx$$
$$\Rightarrow \quad \frac{1}{\sqrt{5}} < I < 1$$
$$\Rightarrow \quad \frac{1}{\sqrt{5}} + \sqrt{3} < I + \sqrt{3} < 1 + \sqrt{3} \Rightarrow [I + \sqrt{3}] = 2$$

Paper II

1. ***(c)*** The given equation reduces to

$$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2 x^2 + 2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]x + 1 = 0$$

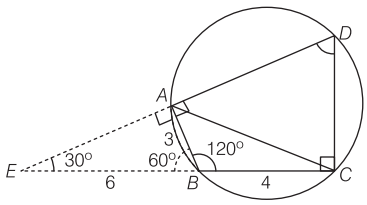
Now, $D = (2[\mathbf{a}\ \mathbf{b}\ \mathbf{c}])^2 - 4[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]^2 = 0$

$\Rightarrow$ The given equation has equal roots.

2. ***(d)*** Let us draw a circle passing through A, B, C and D.

Now, extend CB and DA to meet at E.

Then, ΔECD is a right angled triangle as shown in the figure.



Clearly, $\angle ABE = 60°$ and $\angle BAE = 90°$

$\therefore$ $\angle AEB = 180° - 90° - 60° = 30°$

Now, in ΔAEB

$$\sin 30° = \frac{3}{EB}$$

$$\Rightarrow \quad EB = \frac{3}{\sin 30°} = 6$$

Now, in ΔECD

$$\tan 30° = \frac{CD}{EC} = \frac{CD}{10}$$

$$\Rightarrow \quad CD = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$

3. ***(a)*** Let $x_2 = x_1\ r$, $x_3 = x_1 r^2$ and so $y_2 = y_1 r$, $y_3 = y_1 r^2$, where, r is common ratio.

$$\text{Consider, } \Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} x_1 & y_1 & 1 \\ rx_1 & ry_1 & 1 \\ r^2x_1 & r^2y_1 & 1 \end{vmatrix}$$

$$= x_1 \times y_1 \begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix} = x_1 \times y_1 \times 0 = 0$$

$\Rightarrow$ The given points are collinear.

4. ***(c)*** We have, $\cos\frac{x}{2}\cos\frac{x}{2^2} \ldots \cos\frac{x}{2^n} = \frac{\sin x}{2^n \sin\left(\frac{x}{2^n}\right)}$

On taking log both sides, we get

$$\sum_{k=1}^{n} \log\cos\frac{x}{2^k} = \log\sin x - \log 2^n - \log\sin\left(\frac{x}{2^n}\right)$$

On differentiating both sides w.r.t. x, we get

$$-\left(\sum_{k=1}^{n}\left(\tan\left(\frac{x}{2^k}\right)\cdot\frac{1}{2^k}\right)\right) = \cot x - \cot\left(\frac{x}{2^n}\right)\cdot\frac{1}{2^n}$$

$$\Rightarrow \sum_{k=1}^{n}\left(\tan\left(\frac{x}{2^k}\right)\cdot\frac{1}{2^k}\right) = \frac{1}{2^n}\cdot\cot\left(\frac{x}{2^n}\right) - \cot x$$

Now, for $x \in (0, \pi) - \left\{\frac{\pi}{2}\right\}$,

$$f(x) = \lim_{n\to\infty}\left(\frac{1}{2^n}\cot\left(\frac{x}{2^n}\right) - \cot x\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{2^n}\frac{\frac{x}{2^n}}{\tan\left(\frac{x}{2^n}\right)}\cdot\frac{2^n}{x}\right) - \cot x$$

$$= \frac{1}{x}\lim_{n\to\infty}\left(\frac{\frac{x}{2^n}}{\tan\left(\frac{x}{2^n}\right)}\right) - \cot x = \frac{1}{x} - \cot x$$

$$\therefore \quad f(x) = \begin{cases} \frac{1}{x} - \cot x, & x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} \\ \frac{2}{\pi}, & x = \frac{\pi}{2} \end{cases}$$

Clearly, $\lim\limits_{x\to\frac{\pi}{2}} f(x) = \lim\limits_{x\to\frac{\pi}{2}} \frac{1}{x} - \cot x$

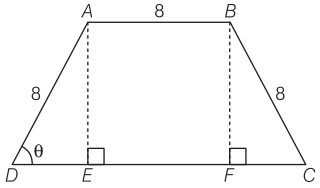
$$= \frac{2}{\pi} = f\left(\frac{\pi}{2}\right)$$

$\therefore$ $f(x)$ is continuous at $x = \frac{\pi}{2}$.

5. ***(b)*** Let $ABCD$ be the given trapezium with

$$AB = BC = AD = 8 \text{ cm}$$

Draw $AE \perp DC$ and $BF \perp DC$



Clearly, area,

$$\Delta = (AB \times AE) + 2\left(\frac{1}{2} \times DE \times AE\right)$$

$$= (8 \times 8\sin\theta) + 8\cos\theta \times 8\sin\theta$$

$$= 64\sin\theta + 64\cos\theta\sin\theta$$

$$\Rightarrow \quad \Delta = 64\sin\theta + 32\sin 2\theta$$

Now, $\frac{d\Delta}{d\theta} = 0$

$$\Rightarrow \quad 64\cos\theta + 64\cos 2\theta = 0$$

$$\Rightarrow \quad \cos\theta + \cos 2\theta = 0$$

$$\Rightarrow \quad 2\cos^2\theta + \cos\theta - 1 = 0$$

$$\Rightarrow \quad (2\cos\theta - 1)(\cos\theta + 1) = 0$$

$$\Rightarrow \quad \cos\theta = \frac{1}{2}$$

[$\because$ in ΔAED, $\theta < 90°$, therefore $\cos\theta \neq 0$]

$$\Rightarrow \quad \theta = \frac{\pi}{3}$$

$$\therefore \quad \Delta_{max} = 64\sin\frac{\pi}{3} + 32\sin\frac{2\pi}{3}$$

$$= 64\sin\frac{\pi}{3} + 32\sin\frac{\pi}{3}$$

$$= 96\sin\frac{\pi}{3} = 96\frac{\sqrt{3}}{2}$$

$$= 48\sqrt{3} \text{ cm}^2$$

6. ***(c)*** If LCM of p, q is $r^2t^4s^2$, then distribution of factor r is as follows

p	r^0	r^1	r^2	r^2	r^2
q	r^2	r^2	r^2	r^1	r^0

Thus, factor r can be distributed in $2 \times 3 - 1$ ways.
Similarly, factors t and s can be distributed in $2 \times 5 - 1$ and $2 \times 3 - 1$ ways, respectively.
Hence, number of ordered pairs

$$(2 \times 3 - 1) \times (2 \times 5 - 1) \times (2 \times 3 - 1) = 225$$

7. ***(a, c, d)*** We have, $(\sin^{-1} x + \sin^{-1} w)(\sin^{-1} y + \sin^{-1} z) = \pi^2$

$$\Rightarrow \quad \sin^{-1} x + \sin^{-1} w = \sin^{-1} y + \sin^{-1} z = \pi$$

or $\sin^{-1}x + \sin^{-1}w = \sin^{-1}y + \sin^{-1}z = -\pi$

$\Rightarrow$ $x = y = z = w = 1$ or $x = y = z = w = -1$

Hence, the maximum value of

$$\begin{vmatrix} x^{N_1} & y^{N_2} \\ z^{N_3} & w^{N_4} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$$

and minimum value $= \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -2$

Also, there are 16 different determinants as each place value is either 1 or –1.

8. ***(a, b, d)*** We have, $f(2-x) = f(2+x)$,

$f(4-x) = f(4+x)$

$\Rightarrow$ $f(4+x) = f(4-x) = f(2+2-x)$

$= f(2-(2-x)) = f(x)$

$\Rightarrow$ 4 is a period of $f(x)$

$$\int_0^{50} f(x)\,dx = \int_0^{48} f(x)\,dx + \int_{48}^{50} f(x)\,dx$$

$$= 12\int_0^4 f(x)\,dx + \int_0^2 f(x)\,dx$$

[in second integral, replacing x by $x + 48$ and then using $f(x) = f(x+48)$]

$$= 12\left(\int_0^2 f(x)\,dx + \int_0^2 f(4-x)\,dx\right) + 5$$

$$= 12\left(\int_0^2 f(x)\,dx + \int_0^2 f(4+x)\,dx\right) + 5$$

$$= 24\int_0^2 f(x)\,dx + 5 = 125$$

$$\int_{-4}^{46} f(x)\,dx = \int_{-4}^{-2} f(x)\,dx + \int_{-2}^{-2+48} f(x)\,dx$$

$$= \int_0^2 f(x+4)\,dx + 12\int_0^2 f(x)\,dx$$

$$= \int_0^2 f(x)\,dx + 24\int_0^2 f(x)\,dx$$

$$= 125$$

Also, $\int_0^{52} f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^{4+48} f(x)\,dx$

$$= \int_0^2 f(4-x)\,dx + 12\int_0^4 f(x)\,dx$$

$$= \int_0^2 f(4+x)\,dx + 24\int_0^2 f(x)\,dx$$

$$= \int_0^2 f(x)\,dx + 24\int_0^2 f(x)\,dx$$

$$= 125$$

$$\int_1^{51} f(x)\,dx = \int_1^3 f(x)\,dx + \int_3^{3+48} f(x)\,dx$$

$$= \int_1^3 f(x)\,dx + 12\int_0^4 f(x)\,dx$$

$$= \int_0^2 f(x+1)\,dx + 24\int_0^2 f(x)\,dx$$

$$\neq 125$$

9. ***(a, b)*** For $b = 1$, we have $f(g(0)) = f(\sin 0° + 1)$

$= f(1) = 1 + a$

Also, $f(g(0^+)) = \lim_{x \to 0^+} f(\sin x + 1) = f(1) = 1 + a$

and $f(g(0^-)) = \lim_{x \to 0^-} f(\{x\}) = f(1^-) = 1 + a$

Hence, $f(g(x))$ is continuous for $b = 1$.

For $b < 0$,

$f(g(0)) = f(\sin 0 + b) = f(b) = 2 - b$

$f(g(0^+)) = \lim_{x \to 0^+} f(\sin x + b) = f(b) = 2 - b$

and $f(g(0^-)) = \lim_{x \to 0^-} f(\{x\}) = f(1) = 1 + a$

For continuous at $x = 0$, we must have

$2 - b = 1 + a$ or $a + b = 1$

10. ***(b, c, d)*** Given equation is $x^3 - ax^2 + bx - 1 = 0$.

If roots of equation are α, β and γ, then

$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2$

$- 2(\alpha\beta + \beta\gamma + \gamma\alpha) = a^2 - 2b$

$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma)$

$= b^2 - 2a$

$\alpha^2\beta^2\gamma^2 = 1$

So, the equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is given by

$x^3 - (a^2 - 2b)x^2 + (b^2 - 2a)x - 1 = 0$

It is identical

$x^3 - ax^2 + bx - 1 = 0$

$\Rightarrow$ $a^2 - 2b = a$ and $b^2 - 2b = b$

Eliminating b, we get

$$\frac{(a^2-a)^2}{4} - 2a = \frac{a^2 - a}{2}$$

$\Rightarrow$ $a\{a(a-1)^2 - 8 - 2(a-1)\} = 0$

$\Rightarrow$ $a(a^3 - 2a^2 - a - 6) = 0$

$\Rightarrow$ $a(a-3)(a^2 + a + 2) = 0$

$\Rightarrow$ $a = 0$ or $a = 3$ or $a^2 + a + 2 = 0$

Which gives $b = 0$ or $b = 3$ or $b^2 + b + 2 = 0$

So, $a = b = 0$ or $a = b = 3$

or a, b are the roots of $x^2 + x + 2 = 0$.

11. ***(b, c)*** We have,

$x^2 + y^2 - 8x - 16y + 60 = 0$ ...(i)

Equation of chord of contact from $(-2, 0)$ is

$-2x - 4(x-2) - 8y + 60 = 0$

$\Rightarrow$ $3x + 4y - 34 = 0$ ...(ii)

Solving Eqs. (i) and (ii), we get

$$x^2 + \left(\frac{34 - 3x}{4}\right)^2 - 8x - 16\left(\frac{34 - 3x}{4}\right) + 60 = 0$$

$\Rightarrow$ $16x^2 + 1156 - 204x + 9x^2 - 128x$

$- 2176 + 192x + 960 = 0$

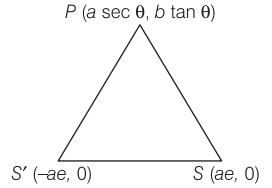
$\Rightarrow$ $5x^2 - 28x - 12 = 0$

$\Rightarrow$ $(x-6)(5x+2) = 0$

$\Rightarrow$ $x = 6, -\frac{2}{5}$

$\Rightarrow$ Points are $(6, 4), \left(-\frac{2}{5}, \frac{44}{5}\right)$.

12. ***(a, b)*** Let (h, k) be excentre, then



$$h = \frac{a(ae\sec\theta + a) - ae(ae\sec\theta - a) - 2ae(a\sec\theta)}{2ae(\sec\theta - 1)}$$

$h = -a \Rightarrow x = -a$ (for $a\sec\theta > 0$)

Similarly, $x = a$ for $a\sec\theta < 0$

$\Rightarrow$ Locus is $x^2 = a^2$.

Again, let (h, k) be excentre opposite to $\angle S'$

$$\therefore \quad h = \frac{2a^2e\sec\theta + a^2e^2\sec\theta + a^2e + a^2e^2\sec\theta - a^2e}{2a + 2ae}$$

$$\Rightarrow \quad h = ae\sec\theta,\ k = \frac{2aeb\tan\theta}{2a + 2ae}$$

$\Rightarrow$ Locus is hyperbola.

13. ***(a, c)*** Let P_1 contains the line $\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}})$, hence contains the point $\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and is normal to vector $(\hat{\mathbf{i}} + \hat{\mathbf{j}})$.

Hence, equation of plane is

$$(\mathbf{r} - (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})) \cdot (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = 0$$

or $$x + y = 2$$

Plane P_2 contains the line

$$\mathbf{r} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}} + \lambda\,(\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}) \text{ and } \hat{\mathbf{j}}$$

Hence, the equation of plane is

$$\begin{vmatrix} x - 0 & y - 1 & z - 0 \\ 1 - 0 & 1 - 1 & 1 - 0 \\ 1 & -1 & -1 \end{vmatrix} = 0 \text{ or } x + 2y - z = 2$$

If θ is the acute angle between P_1 and P_2, then

$$\cos\theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} = \frac{(\hat{\mathbf{i}} + \hat{\mathbf{j}}) \cdot (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})}{\sqrt{2}\sqrt{6}} = \frac{3}{\sqrt{2}\sqrt{6}} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

As, L is contained in $P_2 \Rightarrow \theta = 0°$

14. ***(a, c, d)*** We have, $f(x) = \int_{1/x}^{x} \frac{e^{-\left(t + \frac{1}{t}\right)}}{t}dt$

$$f'(x) = \frac{e^{-\left(x + \frac{1}{x}\right)}}{x} - \frac{e^{-\left(\frac{1}{x} + x\right)}}{\frac{1}{x}}\left(-\frac{1}{x^2}\right)$$

$$= \frac{e^{-\left(x + \frac{1}{x}\right)}}{x} + \frac{e^{-\left(x + \frac{1}{x}\right)}}{x} = \frac{2e^{-\left(x + \frac{1}{x}\right)}}{x}$$

As, $f'(x) > 0\ \forall\ x \in (0, \infty)$

$\therefore$ $f(x)$ is monotonically increasing on $(0, \infty)$.

Now, $f(x) + f\left(\frac{1}{x}\right) = \int_{1/x}^{x} \frac{e^{-\left(t + \frac{1}{t}\right)}}{t}dt + \int_{x}^{1/x} \frac{e^{-\left(t + \frac{1}{t}\right)}}{t}dt$

$$= 0,\ \forall\ x \in (0, \infty)$$

Now, let $g(x) = f(2^x) = \int_{2^{-x}}^{2^x} \frac{e^{-\left(t + \frac{1}{t}\right)}}{t}dt$

$$g(-x) = f(2^{-x}) = \int_{2^x}^{2^{-x}} \frac{e^{-\left(t + \frac{1}{t}\right)}}{t}dt = -g(x)$$

$\therefore$ $f(2^x)$ is an odd function.

Sol. (Q. Nos. 15-16) Let $P(i)$ be the probability that exactly i students are passing an examination. Now given, that $P(A_i) \propto i^2$

$\because$ $P(A_i) = \lambda\, i^2$ [λ is constant]

$$\Rightarrow \quad \sum_{i=1}^{10} P(Ai) = \sum_{i=1}^{10} \lambda i^2 = \lambda\frac{10 \times 11 \times 21}{6} = 1$$

$$\therefore \quad \lambda \cdot 385 = 1 \ \Rightarrow\ \lambda = \frac{1}{385}$$

Let A represents the event that selected students have passed the examination.

$$\therefore \quad P(A) = \sum_{i=1}^{10} P\left(\frac{A}{A_i}\right) P(A_i)$$

$$= \sum_{i=1}^{10} \frac{i^2}{385} \cdot \frac{i}{10} = \frac{1}{3850}\sum_{i=1}^{10} i^3 = \frac{10^2 \times 11^2}{4 \times 3850} = \frac{11}{14}$$

Now, $$P\left(\frac{A_i}{A}\right) = \frac{P\left(\frac{A}{A_i}\right) P(A_i)}{P(A)}$$

$$= \frac{\frac{1}{385} \times \frac{1}{10}}{\frac{11}{14}}$$

$$= \frac{1}{11 \times 55} \times \frac{1}{5} = \frac{1}{3025}$$

15. ***(c)*** **16.** ***(a)***

Sol. (Q. Nos. 17-18) We have,

$$f(x) = \begin{vmatrix} a^2 + x & ab & ac \\ ab & b^2 + x & bc \\ ac & bc & c^2 + x \end{vmatrix}$$

Multiplying C_1 by a, we get

$$f(x) = \frac{1}{a}\begin{vmatrix} a^3 + ax & ab & ac \\ a^2b & b^2 + x & bc \\ a^2c & bc & c^2 + x \end{vmatrix}$$

Applying $C_1 \to C_1 + bC_2 + cC_3$ and taking $a^2 + b^2 + c^2 + x$ common, we get

$$f(x) = \frac{1}{a}(a^2 + b^2 + c^2 + x)\begin{vmatrix} a & ab & ac \\ b & b^2 + x & bc \\ c & bc & c^2 + x \end{vmatrix}$$

Applying $C_2 \to C_2 - bC_1$ and $C_3 \to C_3 - cC_1$, we get

$$f(x) = \frac{1}{a}(a^2 + b^2 + c^2 + x)\begin{vmatrix} a & 0 & 0 \\ b & x & 0 \\ c & 0 & x \end{vmatrix}$$

$$f(x) = \frac{1}{a}(a^2 + b^2 + c^2 + x)ax^2$$

$$= x^2(a^2 + b^2 + c^2 + x)$$

$\Rightarrow$ $f(x) = x^2(a^2 + b^2 + c^2 + x)$

If $f(x) = 0 \Rightarrow x = 0, -(a^2 + b^2 + c^2)$

$$f'(x) = (a^2 + b^2 + c^2)\ 2x + 3x^2 = 0$$

$\Rightarrow$ $x = 0, x = -\frac{2}{3}(a^2 + b^2 + c^2)$

Clearly, in question 17 option (d) is correct.

$$f''(x) = 2(a^2 + b^2 + c^2) + 6x$$

Clearly $f''(x) > 0$ for $x = 0$

$\therefore$ Local minimum value of $f(x)$ is zero.

17. ***(d)*** **18.** ***(d)***

CONCEPT MAP

your Revision Tool

LIMIT, CONTINUITY AND DIFFERENTIABILITY

1 LEFT HAND LIMIT (LHL) AND RIGHT HAND LIMIT (RHL)

For a function $f(x)$ at $x = a$,

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a-h)$$

and $$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a+h)$$

If LHL = RHL, then we say that $\lim_{x \to a} f(x)$ exists.

2 SOME USEFUL RESULTS ON LIMIT

Let $\lim_{x \to a} f(x) = l$ and $\lim_{x \to a} g(x) = m$

If l and m both exist, then

(i) $\lim_{x \to a} (f \pm g)(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
$= l \pm m$

(ii) $\lim_{x \to a} (fg)(x) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = lm$

(iii) $\lim_{x \to a} \left(\frac{f}{g}\right)(x) = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{l}{m}$,
provided $m \neq 0$

(iv) $\lim_{x \to a} kf(x) = k \cdot \lim_{x \to a} f(x)$, where k is constant.

(v) $\lim_{x \to a} |f(x)| = \left|\lim_{x \to a} f(x)\right| = |l|$

(vi) $\lim_{x \to a} \{f(x)\}^{g(x)} = l^m$

(vii) If $f(x) \leq g(x)$ for every x in the deleted neighbourhood of a, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$.

(viii) **Sandwich theorem** If $f(x) \leq g(x) \leq h(x)$ for every x in the deleted neighbourhood of a and $\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$, then $\lim_{x \to a} g(x) = l$.

(ix) $\lim_{x \to a} fog(x) = f\left(\lim_{x \to a} g(x)\right) = f(m)$

In particular,

(a) $\lim_{x \to a} \log f(x) = \log\left(\lim_{x \to a} f(x)\right) = \log l$

(b) $\lim_{x \to a} e^{f(x)} = e^{\lim_{x \to a} f(x)} = e^l$

(x) If $\lim_{x \to a} f(x) = +\infty$ or $-\infty$, then $\lim_{x \to a} \frac{1}{f(x)} = 0$

3 INDETERMINATE FORMS

$\frac{\infty}{\infty}, \frac{0}{0}, \infty - \infty, 0 \times \infty, 1^\infty, 0^0, \infty^0$ are known as indeterminate forms.

4 EVALUATION OF LIMITS

In order to evaluate algebraic limits, we use the following methods:

(A) BY FACTORISATION METHOD

If by putting $x = a$, the rational function $\frac{f(x)}{g(x)}$ takes the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $(x - a)$ is a factor of both $f(x)$ and $g(x)$.

Therefore, to evaluate $\lim_{x \to a} \frac{f(x)}{g(x)}$, we first factorise the numerator and denominator and then cancel out the common factor $(x - a)$. After cancelling out the common factor $(x - a)$, we again put $x = a$ in the given expression and see whether we get a meaningful number or not. This process is repeated till we get a meaningful number.

(B) BY RATIONALISATION METHOD

This method is particularly used when either numerator or denominator or both involve expressions consisting of square roots. If an expression involving square root vanishes when x is replaced by the value towards which it approaches, then we rationalise that expression.

(C) BY USING STANDARD RESULTS

If an expression reduces to the form $\frac{x^n - a^n}{x - a}$, where $n \in Q$, then $\lim_{x \to a} \frac{x^n - a^n}{x - a}$ can be evaluated by using the formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$.

(D) WHEN LIMIT TENDS TO ∞ OR $-\infty$

To evaluate this type of limits, we follow the following steps:

***Step* I** Write down the given expression in the form of a rational function, i.e. $\frac{f(x)}{g(x)}$, if it is not so.

***Step* II** If k is the highest power of x in numerator and denominator both, then divide each term in numerator and denominator by x^k.

***Step* III** Use the result $\lim_{x \to \infty} \frac{1}{x^n} = 0$, where $n > 0$.

DIFFERENTIABILITY IN A SET

A function $f(x)$ defined on $[a,b]$ is said to be differentiable or derivable at the end points a and b, if it is differentiable from the right at a and from the left at b.

In other words, $\lim_{x \to a^+} \frac{f(x) - f(a)}{x - a}$

and $\lim_{x \to b^-} \frac{f(x) - f(b)}{x - b}$ both exist.

If f is derivable in the open interval (a,b) and also at the end points a and b, then f is said to be derivable in the closed interval $[a,b]$. A function f is said to be a differentiable function, if it is differentiable at every point of its domain.

If a function is differentiable at each $x \in R$, then it is said to be everywhere differentiable. e.g. A constant

An important Result If m, n are positive integers and $a_0, b_0 \neq 0$ are non-zero real numbers, then

$$\lim_{x \to \infty} \frac{a_0 x^m + a_1 x^{m-1} + \ldots + a_{m-1} x + a_m}{b_0 x^n + b_1 x^{n-1} + \ldots + b_{n-1} x + b_n}$$

$$= \begin{cases} \frac{a_0}{b_0}, & \text{if } m = n \\ 0, & \text{if } m < n \\ \infty, & \text{if } m > n \text{ and } a_0 b_0 > 0 \\ -\infty, & \text{if } m > n \text{ and } a_0 b_0 < 0 \end{cases}$$

(E) TRIGONOMETRIC LIMITS

To evaluate trigonometric limits, the following results are very useful:

(i) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(ii) $\lim_{x \to 0} \frac{\tan x}{x} = 1$

(iii) $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1$

(iv) $\lim_{x \to 0} \frac{\tan^{-1} x}{x} = 1$

(v) $\lim_{x \to 0} \frac{\sin x^\circ}{x} = \frac{\pi}{180}$

(vi) $\lim_{x \to 0} \cos x = 1$

(vii) $\lim_{x \to a} \frac{\sin(x - a)}{x - a} = 1$

(viii) $\lim_{x \to a} \frac{\tan(x - a)}{x - a} = 1$

(ix) $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

(x) $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3} = \frac{1}{2}$

(F) EXPONENTIAL AND LOGARITHMIC LIMITS

In order to evaluate exponential and logarithmic limits, we use the following formulae:

(i) $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a, a > 0$

function, a polynomial function, sin x, cos x etc., are everywhere differentiable.

Following are some standard results on differentiability:

(i) Every polynomial function is differentiable at each $x \in R$.

(ii) The exponential function $a^x, a > 0$ is differentiable at each $x \in R$.

(iii) Every constant function is differentiable at each $x \in R$.

(iv) The logarithmic function is differentiable at each point in its domain.

(v) Trigonometric and inverse trigonometric functions are differentiable in their domains.

(vi) The sum, difference, product and quotient of two differentiable functions is differentiable.

(vii) The composition of differentiable function is a differentiable function.

(ii) $\lim_{x\to 0} \dfrac{e^x - 1}{x} = 1$

(iii) $\lim_{x\to 0} \dfrac{\log_e (1+x)}{x} = 1$

(iv) $\lim_{x\to 0} \dfrac{\log_a (1+x)}{x} = \dfrac{1}{\log_e a}, a > 0, a \neq 1$

Ⓖ LIMITS OF THE FORM 1^∞

In order to evaluate limits of the form 1^∞, we use the following theorem:

Theorem If $f(x)$ and $g(x)$ are two functions such that

$$\lim_{x\to a} f(x) = 0 = \lim_{x\to a} g(x), \text{ then}$$

$$\lim_{x\to a} \{1 + f(x)\}^{1/g(x)} = e^{\lim_{x\to a} \frac{f(x)}{g(x)}}$$

Or If $\lim_{x\to a} f(x) = 1$ and $\lim_{x\to a} g(x) = \infty$, then

$$\lim_{x\to a} \{f(x)\}^{g(x)} = \lim_{x\to a} [1 + \{f(x) - 1\}]^{g(x)}$$
$$= e^{\lim_{x\to a} \{f(x)-1\} g(x)}$$

Particular Cases

(i) $\lim_{x\to 0} (1+x)^{1/x} = e$ (ii) $\lim_{x\to \infty} \left(1 + \dfrac{1}{x}\right)^x = e$

(iii) $\lim_{x\to 0} (1+\lambda x)^{1/x} = e^{\lambda}$

(iv) $\lim_{x\to \infty} \left(1 + \dfrac{\lambda}{x}\right)^x = e^{\lambda}$

Ⓗ BY USING L' HOSPITAL'S RULE

De' L' Hospital's Rule If $f(x)$ and $g(x)$ are two functions of x such that

(i) $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$

(ii) both are continuous at $x = a$

(iii) both are differentiable at $x = a$

(iv) $f'(x)$ and $g'(x)$ are continuous at the point $x = a$.

Then, $\lim_{x\to a} \dfrac{f(x)}{g(x)} = \lim_{x\to a} \dfrac{f'(x)}{g'(x)}$,

Remark *The above rule is also applicable, if* $\lim_{x\to a} f(x) = \infty$ *and* $\lim_{x\to a} g(x) = \infty$.

Generalisation If $\lim_{x\to a} \dfrac{f'(x)}{g'(x)}$ assumes the indeterminate form $\dfrac{0}{0}$ and $f'(x), g'(x)$ satisfy all the conditions embodied in De' L' Hospital's rule, we can repeat the application of this rule on $\dfrac{f'(x)}{g'(x)}$ to get

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to 0} \frac{f''(x)}{g''(x)}.$$

Remark *Sometimes, it may be necessary to repeat the above process a number of times till our goal of evaluating limit is achieved.*

5 CONTINUITY AT A POINT

A function $f(x)$ is said to be continuous at a point $x = a$ of its domain iff $\lim_{x\to a} f(x) = f(a)$.

Thus, $f(x)$ is continuous at $x = a$.

$\Leftrightarrow \quad \lim_{x\to a} f(x) = f(a)$

$\Leftrightarrow \quad \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$

If $\lim_{x\to a^-} f(x) = f(a)$, then $f(x)$ is continuous from left at $x = a$ and $f(x)$ is continuous from right at $x = a$, if $\lim_{x\to a^+} f(x) = f(a)$.

6 CONTINUITY ON AN OPEN INTERVAL

A function $f(x)$ is said to be continuous on an open interval (a, b) iff it is continuous at every point on the interval (a, b).

7 CONTINUITY ON A CLOSED INTERVAL

A function $f(x)$ is said to be continuous on a closed interval $[a, b]$ iff

(i) f is continuous on the open interval (a, b)

(ii) $\lim_{x\to a^+} f(x) = f(a)$

(iii) $\lim_{x\to b^-} f(x) = f(b)$

In other words, $f(x)$ is continuous on $[a,b]$ iff it is continuous on (a, b) and it is continuous at a from the right and at b from the left.

8 DISCONTINUOUS FUNCTIONS

A function f is said to be discontinuous at a point a of its domain D, if it is not continuous there at. The point a is then called a point of discontinuity of the function. The discontinuity may arise due to any of the following situations:

(i) $\lim_{x\to a^+} f(x)$ or $\lim_{x\to a^-} f(x)$ both may not exists.

(ii) $\lim_{x\to a^+} f(x)$ as well as $\lim_{x\to a^-} f(x)$ may exist, but are unequal.

(iii) $\lim_{x\to a^+} f(x)$ as well as $\lim_{x\to a^-} f(x)$ both may exist, but either of the two or both may not be equal to $f(a)$.

We classify the points of discontinuity according to various situations discussed above.

REMOVABLE DISCONTINUITY

A function f is said to have removable discontinuity at $x = a$, if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ but their common value is not equal to $f(a)$.

Such a discontinuity can be removed by assigning a suitable value to the function f at $x = a$.

Discontinuity of first kind A function f is said to have a discontinuity of the first kind at $x = a$, if $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ both exist but are not equal.

f is said to have a discontinuity of the first kind from the left at $x = a$, if $\lim_{x\to a^-} f(x)$ exists but not equal to $f(a)$. Discontinuity of the first kind from the right is similarly defined.

Discontinuity of second kind A function f is said to have a discontinuity of the second kind at $x = a$, if neither $\lim_{x\to a^-} f(x)$ nor $\lim_{x\to a^+} f(x)$ exists. f is said to have discontinuity of the second kind from the left at $x = a$, if $\lim_{x\to a^-} f(x)$ does not exist.

Similarly, if $\lim_{x\to a^+} f(x)$ does not exist, then f is said to have discontinuity of the second kind from the right at $x = a$.

9 DIFFERENTIABILITY AT A POINT

Let $f(x)$ be a real valued function defined on an open interval (a, b) and $c \in (a, b)$. Then, $f(x)$ is said to be differentiable or derivable at $x = c$ iff $\lim_{x\to c} \dfrac{f(x) - f(c)}{x - c}$ exists finitely.

This limit is called the derivative or differential coefficient of the function $f(x)$ at $x = c$ and is denoted by $f'(c)$ or $Df(c)$ or $\left\{\dfrac{d}{dx} f(x)\right\}_{x=c}$.

Thus, $f(x)$ is differentiable at $x = c$.

$\Leftrightarrow \quad \lim_{x\to c} \dfrac{f(x) - f(c)}{x - c}$ exists finitely.

$\Leftrightarrow \quad \lim_{x\to c^-} \dfrac{f(x) - f(c)}{x - c} = \lim_{x\to c^+} \dfrac{f(x) - f(c)}{x - c}$

$\Leftrightarrow \quad \lim_{h\to 0} \dfrac{f(c-h) - f(c)}{-h} = \lim_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$

$\lim_{x\to c^-} \dfrac{f(x) - f(c)}{x - c}$ or $\lim_{h\to 0} \dfrac{f(c-h) - f(c)}{-h}$

is called the left hand derivative of $f(x)$ at $x = c$ and is denoted by $f'(c^-)$ or $Lf'(c)$, while

$\lim_{x\to c^+} \dfrac{f(x) - f(c)}{x - c}$ or $\lim_{h\to 0} \dfrac{f(c+h) - f(c)}{h}$

is called the right hand derivative of $f(x)$ at $x = c$ and is denoted by $f'(c^+)$ or $Rf'(c)$.

Thus, $f(x)$ is differentiable at $x = c$
$\Leftrightarrow Lf'(c) = Rf'(c)$.

If $Lf'(c) \neq Rf'(c)$, then we say that $f(x)$ is not differentiable at $x = c$.

TARGET JEE 2017

TEST RIDER

Comprehensive Simulator Test Series for JEE Main & Advanced

JEE ADVANCED

(PAPER TWO PREP UP)

Mock Questions from Complete Syllabus with Hints and Answers

FULL TEST 2

(COMPLETE SYLLABUS) WITH HINTS AND ANSWERS

PAPER I

SECTION 1 (MM : 18)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. If n is selected from the set $\{1, 2, 3, \ldots, 10\}$ and the number $2^n + 3^n + 5^n$ is formed, then the total number of ways of selecting n so that the formed number is divisible by 4, is
(a) 50 *(b)* 49 *(c)* 48 *(d)* None of these

2. The base of prism is equilateral triangle. If the distance from the centre of one base to one of the vertices of the other base is l, then altitude of the prism for which the volume is greatest, is
(a) $\frac{l}{2}$ *(b)* $\frac{l}{\sqrt{3}}$ *(c)* $\frac{l}{3}$ *(d)* $\frac{l}{4}$

3. The value of k for which the inequality $k\cos^2 x - k\cos x + 1 \geq 0, \forall\ x \in (-\infty, \infty)$ holds, is
(a) $k < \frac{-1}{2}$ *(b)* $k > 4$ *(c)* $\frac{-1}{2} \leq k \leq 4$ *(d)* $\frac{1}{2} \leq k \leq 5$

4. If the roots of $x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in GP and the sum of their reciprocal is 10, then $|S|$ equals
(a) 4 *(b)* 6 *(c)* 8 *(d)* 32

5. An artillery target may be either at point I with probability 8/9 or at point II with probability 1/9. We have 55 shells, each of which can be fired either at point I or II. Each shell may hit the target, independent of the other shells, with probability 1/2. The maximum number of shells must be fired at point I to have maximum probability is
(a) 20 *(b)* 25 *(c)* 29 *(d)* 35

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

6. Let $y(x)$ be a solution of the differential equation $(1+e^x)\,y' + ye^x = 1$. If $y(0) = 2$, then which of the following statement(s) is/are true?
(a) $y(-4) = 0$
(b) $y(-2) = 0$
(c) $y(x)$ has a critical point in the interval $(-1, 0)$
(d) $y(x)$ has no critical point in the interval $(-1, 0)$

7. If the equations of two equal sides AB and AC of an isosceles ΔABC are $x + y = 5$ and $7x - y = 3$ respectively, then the equation of side BC, if ar $(\Delta ABC) = 5$ sq units, is
(a) $x - 3y + 1 = 0$ *(b)* $x - 3y + 21 = 0$
(c) $3x + y + 2 = 0$ *(d)* $3x + y - 12 = 0$

8. In ΔABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2 A/2$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then
(a) $b + c = 4a$
(b) $b + c = 2a$
(c) locus of point A is an ellipse
(d) locus of point A is a pair of straight lines

9. If point M is moved on the circle $(x-4)^2 + (y-8)^2 = 20$, then it broke away from it and moving along a tangent to the circle cut the X-axis at point $(-2, 0)$. The coordinates of the point on the circle at which the moving point broke away, is
(a) $\left(\frac{42}{5}, \frac{36}{5}\right)$ *(b)* $\left(\frac{-2}{5}, \frac{44}{5}\right)$
(c) $(6, 4)$ *(d)* $(2, 4)$

10. The equation of the line that touches the curves $y = x|x|$ and $x^2 + (y-2)^2 = 4$, where $x \neq 0$, is
(a) $y = 4\sqrt{5}x + 20$ *(b)* $y = 4\sqrt{3}x - 12$
(c) $y = 0$ *(d)* $y = -4\sqrt{5}x - 20$

11. If $f(x) = \begin{cases} \left(\frac{3}{2}\right)^{(\cot 3x)/(\cot 2x)}, & 0 < x < \frac{\pi}{2} \\ b + 3, & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{(a|\tan x|)/b}, & \frac{\pi}{2} < x < \pi \end{cases}$

is continuous at $x = \frac{\pi}{2}$, then
(a) $a = 0$ *(b)* $a = 2$ *(c)* $b = -2$ *(d)* $b = 2$

12. If $f(x-y)$, $f(x)\,f(y)$ and $f(x+y)$ are in AP, for all x, y and $f(0) \neq 0$, then
(a) $f(4) = f(-4)$ *(b)* $f(2) + f(-2) = 0$
(c) $f'(4) + f'(-4) = 0$ *(d)* $f'(2) = f'(-2)$

13. Let M be a 2×2 symmetric matrix with integer entries. Then, M is invertible, if
(a) the first column of M is the transpose of the second row of M
(b) the second row of M is the transpose of the first column of M
(c) M is a diagonal matrix with non-zero entries in the main diagonal
(d) the product of entries in the main diagonal of M is not the square of an integer

SECTION 3 (MM : 12)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 In all other cases.

14. Let $a = 3^{\frac{1}{233}} + 1$ and for all $n \geq 3$, let $f(n) = {}^nC_0 \cdot a^{n-1} - {}^nC_1 \cdot a^{n-2} + {}^nC_2 \cdot a^{n-3} - \cdots + (-1)^{n-1} \cdot {}^nC_{n-1} \cdot a^0$. If the value of $f(2007) + f(2008) = 3^k$, where $k \in N$, then the value of k is ..... .

15. If $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$, then the value of $(\Delta_{\max})/2$ is ..... .

16. For any integer k, let $\alpha_k = \cos\frac{k\pi}{7} + i\sin\frac{k\pi}{7}$, where $i = \sqrt{-1}$. Then, the value of the expression $\frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|}$ is ..... .

17. If $F(x) = \frac{1}{x^2}\int_4^x [4t^2 - 2F'(t)]\,dt$, then $(9F'(4))/4$ is ... .

18. The largest value of non-negative integer a for which $\lim_{x \to 1}\left\{\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$ is ..... .

PAPER II

SECTION 1 (MM : 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. Consider three matrices $A=\begin{bmatrix}2 & 1\\ 4 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & 4\\ 2 & 3\end{bmatrix}$ and $C=\begin{bmatrix}3 & -4\\ -2 & 3\end{bmatrix}$. Then, the value of the sum of

$$\text{tr}(A)+\text{tr}\left(\frac{ABC}{2}\right)+\text{tr}\left(\frac{A(BC)^2}{4}\right)+\text{tr}\left(\frac{A(BC)^3}{8}\right)+\ldots, \text{ is}$$

(a) 6 *(b)* 9
(c) 12 *(d)* None of these

2. The area of the region bounded by $x=0,\ y=0,\ x=2,\ y=2,\ y\le e^x$ and $y\ge \ln x$ is
(a) $(6-4\ln 2)$ sq units
(b) $(4\ln 2-2)$ sq units
(c) $(2\ln 2-4)$ sq units
(d) $(6-2\ln 2)$ sq units

3. In ΔABC, if $\tan A:\tan B:\tan C=3:4:5$, then the value of $\sin A\sin B\sin C$ is
(a) $\frac{2}{\sqrt{5}}$ *(b)* $\frac{2\sqrt{5}}{7}$ *(c)* $\frac{2\sqrt{5}}{9}$ *(d)* $\frac{2}{3\sqrt{5}}$

4. ABC is a right angled triangle in which $\angle B=90^\circ$ and $BC=a$. If n points $L_1, L_2, \ldots, L_n$ on AB is divided in $(n+1)$ equal parts and $L_1M_1, L_2M_2, \ldots, L_nM_n$ are line segments parallel to BC and $M_1, M_2, \ldots, M_n$ are on AC, then the sum of the lengths of $L_1M_1, L_2M_2, \ldots, L_nM_n$ is
(a) $\frac{a(n+1)}{2}$ *(b)* $\frac{a(n-1)}{2}$ *(c)* $\frac{an}{2}$ *(d)* None of these

5. If $f(x)=\begin{cases}0, \text{ where } x=\dfrac{n}{n+1};\ n=1,2,3,\ldots\\ 1, \text{ elsewhere}\end{cases}$

Then, the value of $\int_0^2 f(x)dx$ is
(a) 1 *(b)* 0 *(c)* 2 *(d)* ∞

6. If perpendiculars are drawn from points on the line $\frac{x+2}{2}=\frac{y+1}{-1}=\frac{z}{3}$ to the plane $x+y+z=3$, then the feet of perpendiculars lie on the line
(a) $\frac{x}{5}=\frac{y-1}{8}=\frac{z-2}{-13}$ *(b)* $\frac{x}{2}=\frac{y-1}{3}=\frac{z-2}{-5}$
(c) $\frac{x}{4}=\frac{y-1}{2}=\frac{z-2}{-7}$ *(d)* $\frac{x}{2}=\frac{y-1}{-7}=\frac{z-2}{5}$

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

7. Let $f(x)=\frac{[x]+1}{\{x\}+1}$ for $f:\left[0,\frac{5}{2}\right)\to\left(\frac{1}{2},3\right]$, where $[\cdot]$ represents the greatest integer function and $\{\cdot\}$ represents the fractional part of x. Then, which of the following is/are true?
(a) $f(x)$ is injective discontinuous function
(b) $f(x)$ is surjective non-differentiable function
(c) $\min\left(\lim_{x\to 1^-} f(x),\ \lim_{x\to 1^+} f(x)\right)=f(1)$
(d) max (x values of point of discontinuity)$=f(1)$

8. Let $f, g:[-1,2]\to R$ be continuous functions which are twice differentiable on the interval $(-1,2)$. Let

the values of f and g at the points -1, 0 and 2 be as given in the following table.

	$x = -1$	$x = 0$	$x = 2$
$f(x)$	3	6	0
$g(x)$	0	1	-1

In each of the intervals $(-1, 0)$ and $(0, 2)$, the function $(f - 3g)''$ never vanishes. Then, the correct statement(s) is/are

(a) $f'(x) - 3g'(x) = 0$ has exactly three solutions in $(-1, 0) \cup (0, 2)$

(b) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$

(c) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(0, 2)$

(d) $f'(x) - 3g'(x) = 0$ has exactly one solution in $(-1, 0)$ and exactly two solutions in $(0, 2)$

9. The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at x equals

(a) -2 (b) $\frac{-2}{3}$ (c) 2 (d) $\frac{2}{3}$

10. The circle(s) touching X-axis at a distance of 3 units from the origin and having an intercept of length $2\sqrt{7}$ on the Y-axis is/are

(a) $x^2 + y^2 - 6x + 8y + 9 = 0$

(b) $x^2 + y^2 - 6x + 7y + 9 = 0$

(c) $x^2 + y^2 - 6x - 8y + 9 = 0$

(d) $x^2 + y^2 - 6x - 7y + 9 = 0$

11. If $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where the inverse trigonometric functions take only the principal values, then the correct options(s) is/are

(a) $\cos\beta > 0$ (b) $\sin\beta < 0$

(c) $\cos(\alpha + \beta) > 0$ (d) $\cos\alpha < 0$

12. If $\sqrt{5 - 12i} + \sqrt{-5 - 12i} = z$, then the principal value of arg (z) can be

(a) $\frac{-\pi}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{3\pi}{4}$ (d) $\frac{-3\pi}{4}$

13. If A is the area and $2s$ is the sum of the sides of a triangle, then

(a) $A \le \frac{s^2}{4}$ (b) $A \le \frac{s^2}{3\sqrt{3}}$

(c) $A < \frac{s^2}{\sqrt{3}}$ (d) None of these

14. Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha\ ?$$

(a) -4 (b) 9

(c) -9 (d) 4

SECTION 3 (MM : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : + 3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 In all other cases.

Passage 1 (Q. Nos. 15-16) The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at points A and B.

15. The equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(a) $2x - \sqrt{5}y - 20 = 0$

(b) $2x - \sqrt{5}y + 4 = 0$

(c) $3x - 4y + 8 = 0$

(d) $4x - 3y + 4 = 0$

16. The equation of the circle with AB as its diameter is

(a) $x^2 + y^2 - 12x + 24 = 0$

(b) $x^2 + y^2 + 12x + 24 = 0$

(c) $x^2 + y^2 + 24x - 12 = 0$

(d) $x^2 + y^2 - 24x - 12 = 0$

Passage 2 (Q. Nos. 17-18) In ΔABC, $\cos A \cos B \cos C = \frac{\sqrt{3} - 1}{8}$ and $\sin A \sin B \sin C = \frac{3 + \sqrt{3}}{8}$.

17. The value of $\tan A + \tan B + \tan C$ is

(a) $\frac{3 + \sqrt{3}}{\sqrt{3} - 1}$ (b) $\frac{\sqrt{3} + 4}{\sqrt{3} - 1}$

(c) $\frac{6 - \sqrt{3}}{\sqrt{3} - 1}$ (d) $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - 1}$

18. The value of $\tan A \tan B + \tan B \tan C + \tan C \tan A$ is

(a) $5 - 4\sqrt{3}$ (b) $5 + 4\sqrt{3}$

(c) $6 + \sqrt{3}$ (d) $6 - \sqrt{3}$

Answers with Hints

Paper I

1. ***(b)*** If n is odd, then

$3^n = 4\lambda_1 - 1$ and $5^n = 4\lambda_2 + 1$

$\Rightarrow \quad 2^n + 3^n + 5^n$ is divisible by 4, if $n \geq 2$.

Thus, $n = 3, 5, 7, 9, \ldots, 99$, i.e. n can take 49 different values.

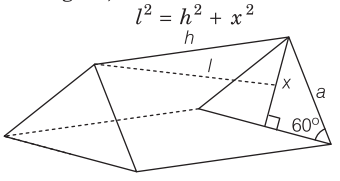
If n is even, then $3^n = 4\lambda_1 + 1$ and $5^n = 4\lambda_2 + 1$

$\Rightarrow 2^n + 3^n + 5^n$ is not divisible by 4 as $2^n + 3^n + 5^n$ will be in the form of $4\lambda + 2$.

Thus, the total number of ways of selecting n is equal to 49.

2. ***(b)*** From the figure,

$l^2 = h^2 + x^2$



Area of base (equilateral triangle) $= \frac{\sqrt{3}}{4}a^2$

Also, $\frac{3x}{2} = a \sin 60° = \frac{\sqrt{3}}{2}a$

$\therefore \quad a = \sqrt{3}\, x$

Now, volume (V) = Height × Area of base

$= \frac{3\sqrt{3}}{4}(hl^2 - h^3)$

$\therefore \quad \frac{dV}{dH} = \frac{3\sqrt{3}}{4}(l^2 - 3h^2)$

$\because \quad \frac{dV}{dh} = 0 \Rightarrow h = \frac{l}{\sqrt{3}}$, which is point of maxima.

3. ***(c)*** We have, $k\cos^2 x - k\cos x + 1 \geq 0, \forall\, x \in (-\infty, \infty)$

$\Rightarrow \quad k(\cos^2 x - \cos x) + 1 \geq 0 \qquad \ldots(i)$

But $\cos^2 x - \cos x = \left(\cos x - \frac{1}{2}\right)^2 - \frac{1}{4}$

$\Rightarrow \quad \frac{-1}{4} \leq \cos^2 x - \cos x \leq 2$

Now, from Eq. (i), we get $2k + 1 \geq 0 \Rightarrow k \geq \frac{-1}{2}$

Also, $\frac{-k}{4} + 1 \geq 0 \Rightarrow k \leq 4$

$\therefore \quad \frac{-1}{2} \leq k \leq 4$

4. ***(d)*** Given, roots of the equation

$x^5 - 40x^4 + Px^3 + Qx^2 + Rx + S = 0$ are in GP.

Let roots be a, ar, ar^2, ar^3 and ar^4.

$\therefore\ a + ar + ar^2 + ar^3 + ar^4 = 40$

and $\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} + \frac{1}{ar^4}$

$= \frac{1 + r + r^2 + r^3 + 4^4}{ar^4} = 10$

$\Rightarrow \quad a^2r^4 = 4 \Rightarrow ar^2 = \pm 2$

Now, S = product of roots

$= a^5r^{10} = (ar^2)^5 = (\pm 2)^5 = \pm 32$

$\therefore \quad |S| = 32$

5. ***(c)*** Let A denotes the event that target is hit when x shells are fired at point I. Let P_1 and P_2 denote the events that the target is at point I and point II, respectively.

Then, $P(P_1) = \frac{8}{9}, P(P_2) = \frac{1}{9}, P\left(\frac{A}{P_1}\right) = 1 - \left(\frac{1}{2}\right)^x$

and $P\left(\frac{A}{P_2}\right) = 1 - \left(\frac{1}{2}\right)^{55-x}$

Now, from total probability theorem,

$P(A) = P(P_1)\, P\left(\frac{A}{P}\right) + P(P_2)\, P\left(\frac{A}{P_2}\right)$

$= \frac{1}{9}\left[8 - 8\left(\frac{1}{2}\right)^x + 1 - \left(\frac{1}{2}\right)^{55-x}\right]$

$= \frac{1}{9}\left[9 - 8\left(\frac{1}{2}\right)^x - \left(\frac{1}{2}\right)^{55-x}\right]$

Now, $\frac{dP(A)}{dx} = \frac{1}{9}\left[-8\left(\frac{1}{2}\right)^x \ln\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^{55-x} \ln\left(\frac{1}{2}\right)\right]$

$= \frac{1}{9}\ln\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{55-x}\left[1 - \left(\frac{1}{2}\right)^{2x-58}\right]$

> 0, if $x < 29 < 0$, if $x > 29$

Therefore, $P(x)$ is maximum at $x = 29$.

Thus, 29 shells must be fired at point I.

6. ***(a, c)*** We have, $(1 + e^x)\frac{dy}{dx} + ye^x = 1$

$\Rightarrow \frac{dy}{dx} + \frac{e^x}{1 + e^x}y = \frac{1}{1 + e^x} \Rightarrow y \cdot (1 + e^x) = \int 1\, dx$

or $\quad (1 + e^x)\, y = x + C$

Given, $x = 0$ and $y = 2 \Rightarrow C = 4$

$\therefore \quad (1 + e^x)\, y = x + 4$

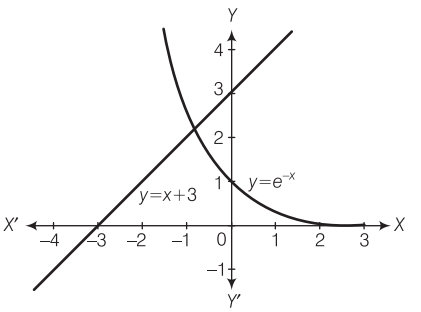
$\Rightarrow \quad y = \frac{x + 4}{e^x + 1}$

$\Rightarrow \quad y(-4) = 0$ and $y(-2) = \frac{2}{e^{-2} + 1}$

$\therefore \frac{dy}{dx} = \frac{(e^x + 1)\cdot 1 - (x + 4)e^x}{(e^x + 1)^2} = \frac{e^x(-x - 3) + 1}{(e^x + 1)^2}$

When $\frac{dy}{dx} = 0$, then $x + 3 = e^{-x}$

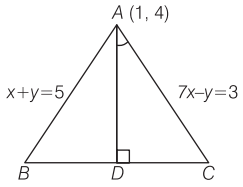
Graphs of $y = x + 3$ and $y = e^{-x}$ are as shown in the following figure.



Clearly, graphs intersect for $x \in (-1, 0)$.

Hence, $y(x)$ has a critical point in the interval $(-1, 0)$.

7. ***(a, b, c, d)*** Side BC will be perpendicular to the bisector of $\angle BAC$.



Now, the equations of the bisectors of lines AB and AC are

$$\frac{(x + y - 5)}{\sqrt{2}} = \pm \frac{(7x - y - 3)}{5\sqrt{2}}$$

$\Rightarrow$ $x - 3y + 11 = 0$ or $3x + y - 7 = 0$

Let the equation of side BC be $x - 3y + k = 0$ and the altitude through vertex A be AD.

Then, the equation of AD is $3x + y - 7 = 0$.

If $AD = \lambda$, then

$$\Delta ABC = \frac{1}{2} \times |BC| = \frac{1}{2} \times \lambda \times 2\lambda |\tan\theta| = \lambda^2 |\tan\theta|$$

Hence, $\lambda^2 \times \frac{1}{2} = 5 \Rightarrow \lambda^2 = 10$

Also $\lambda^2 = \left(\frac{1 - 12 + k}{\sqrt{10}}\right)^2 \Rightarrow 10 = \frac{(k-11)^2}{10}$

$\Rightarrow (k-11)^2 = 100 \Rightarrow k - 11 = \pm 10 \Rightarrow k = 1, 21$

Hence, the equation of BC is $x - 3y + 1 = 0$ or $x - 3y + 21 = 0$.

Similarly, if the equation of BC is $3x + y + k = 0$, then the equation of AD will be $x - 3y + 11 = 0$.

$$\therefore \quad |\tan\theta| = \left|\frac{7 - \left(\frac{1}{3}\right)}{1 + \left(\frac{7}{3}\right)}\right| = 2$$

$$\Rightarrow \quad \lambda^2 |\tan\theta| = 2\lambda^2 = 5 \Rightarrow \lambda^2 = \frac{5}{2}$$

Also, $\frac{5}{2} = \frac{(3 + 4 + k)^2}{10} \Rightarrow 7 + k = \pm 5 \Rightarrow k = 2, -12$

Hence, the equation of BC is $3x + y + 2 = 0$ or $3x + y - 12 = 0$.

Finally, there are four possible equations of side BC, *viz.*, $x - 3y + 1 = 0, x - 3y - 21 = 0, 3x + y + 2 = 0$ and $3x + y - 12 = 0$.

8. ***(b, c)*** We have, $\cos B + \cos C = 4\sin\frac{A}{2}$

$$\Rightarrow 2\cos\left(\frac{B + C}{2}\right)\cos\left(\frac{B - C}{2}\right) = 4\sin^2\frac{A}{2}$$

$$\Rightarrow \quad 2\cos\left(\frac{B - C}{2}\right) = 4\sin\frac{A}{2} \quad \left[\because \cos\left(\frac{B + C}{2}\right) = \frac{\sin A}{2}\right]$$

$$\Rightarrow \quad 2\cos\frac{A}{2}\cos\left(\frac{B - C}{2}\right) = 4\sin\frac{A}{2}\cos\frac{A}{2}$$

$$\Rightarrow \quad 2\sin\left(\frac{B + C}{2}\right)\cos\left(\frac{B - C}{2}\right) = 2\sin A$$

$$\Rightarrow \quad \sin B + \sin C = 2\sin A$$

$$\Rightarrow \quad b + c = 2a \qquad \text{[using sine rule]}$$

Thus, sum of two variable sides b and c is constant '$2a$'.

So, locus of vertex A is ellipse with vertices B and C as its foci.

9. ***(b, c)*** We have, $x^2 + y^2 - 8x - 16y + 60 = 0$ ...(i)

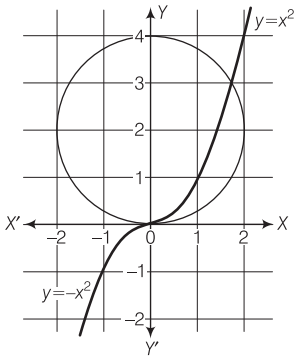
Equation of chord of contact from $(-2, 0)$ is

$$-2x - 4(x - 2) - 8y + 60 = 0$$

$$\Rightarrow \quad 3x + 4y - 34 = 0 \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get the points are $(6, 4)$ and $\left(\frac{-2}{5}, \frac{44}{5}\right)$.

10. ***(a, b, c)*** We have, $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$



Clearly, $y = 0$ is one of the common tangents.

Let the line be $y = mx + c$ ...(i)

If it touches $y = x^2, x \geq 0$, then $mx + c = x^2$ has equal roots.

$$\therefore \quad m^2 + 4c = 0 \text{ or } c = \frac{-m^2}{4} \qquad \text{...(ii)}$$

Also, the line touches the circle.

$$\therefore \quad \frac{|0 - 2 + c|}{\sqrt{m^2 + 1}} = 2 \qquad \text{...(iii)}$$

$$\Rightarrow \quad 4 - 4c + c^2 = 4(1 - 4c)$$

$$\Rightarrow \quad c^2 + 12c = 0$$

$$\Rightarrow \quad c = 0 \text{ or } -12$$

For $c = 0, m = 0$

$\therefore$ The equation of tangent is $y = 0$ for $c = -12, m = \sqrt{48}$.

Therefore, the equation of tangent is $y = \sqrt{48}\,x - 12$.

If line (i) touches $y = -x^2, x < 0$, then the equation $mx + c = -x^2$ has equal roots.

$$\therefore \quad D = 0 \Rightarrow m^2 - 4c = 0 \qquad \text{...(iv)}$$

From Eqs. (iii) and (iv), we get

$$c^2 - 20c = 0$$

$$\therefore \quad c = 0 \text{ or } c = 20$$

For $x = 20, m = \sqrt{80}$

Therefore, the equation of tangent is

$$y = \sqrt{80}\,x + 20$$

11. ***(a, c)*** Here, $f\left(\frac{\pi^-}{2}\right) = \lim_{h \to 0}\left(\frac{3}{2}\right)^{\cot\left\{3\left(\frac{\pi}{2} - h\right)\right\} \Big/ \cot\left\{2\left(\frac{\pi}{2} - h\right)\right\}}$

$$= \lim_{h \to 0}\left(\frac{3}{2}\right)^{\frac{\tan 3h}{-\cot 2h}} = \lim_{h \to 0}\left(\frac{3}{2}\right)^{(-\tan 3h)(\tan 2h)} = 1$$

and $f\left(\frac{\pi^+}{2}\right) = \lim_{h \to 0}\left[1 + \left|\cot\left(\frac{\pi}{2} + h\right)\right|\right]^{\left[a\left|\tan\left(\frac{\pi}{2} + h\right)\right|\right] \Big/ b}$

$$= \lim_{h \to 0}(1 + \tan h)^{\frac{a\cot h}{b}}$$

$$= e^{\lim_{h \to 0}(1 + \tan h - 1)\frac{a\cot h}{b}} = e^{a/b}$$

Also, $f\left(\frac{\pi}{2}\right) = b + 3$

Since, $f(x)$ is continuous at $x = \frac{\pi}{2}$.

$\therefore \quad 1 = b + 3 = e^{a/b} \Rightarrow b = -2$ and $a = 0$

12. ***(a, c)*** Since, $f(x - y)$, $f(x)$ $f(y)$ and $f(x + y)$ are in AP.
Therefore, $f(x + y) + f(x - y) = 2f(x)\ f(y)$ for all x, y.
On putting $x = 0, y = 0$ in Eq. (i), we get

$$f(0) + f(0) = 2f(0)\ f(0)$$

$\Rightarrow \quad f(0) = 1 \qquad [\because f(0) \neq 0]$

On putting $x = 0, y = x$ in Eq. (i), we get

$$f(x) + f(-x) = 2f(0)\ f(x)$$

$\Rightarrow \quad f(x) = f(-x) \qquad \ldots(ii)$

$\Rightarrow \quad f(4) = f(-4)$ and $f(3) = f(-3)$

On differentiating Eq. (ii) w.r.t. x, we get

$$f'(x) + f'(-x) = 0$$

$\Rightarrow \quad f'(4) + f'(-4) = 0$

13. ***(c, d)*** Let $M = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$, where $a, b, c \in I$

(a) If the first column of M is the transpose of the second row of M, then $\begin{bmatrix} a \\ c \end{bmatrix} = \begin{bmatrix} c \\ b \end{bmatrix}$

$\therefore \quad a = b = c$
Thus, $\det(M) = ab - c^2 = 0$
Hence, M is not invertible.

(b) If the second row of M is the transpose of the first column of M, then $\begin{bmatrix} c \\ b \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$

$\therefore \quad a = b = c$
Thus, $\det(M) = ab - c^2 = 0$
Hence, M is not invertible.

(c) If $M = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $a, b \neq 0$

Then, $\det(M) = ab \neq 0$
Hence, M is invertible.

(d) If product of elements in main diagonal which (ab) is not perfect square, then
$\det(M) = ab - c^2 \neq 0$
Hence, M is invertible.

14. ***(9)*** We have, $f(n) = {}^nC_0a^{n-1} - {}^nC_1a^{n-2} + {}^nC_2a^{n-3} - \ldots + (-1)^{n-1} \cdot {}^nC_{n-1}a^0$

$$= \frac{1}{a}({}^nC_0a^n - {}^nC_1\,a^{n-1} + {}^nC_2a^{n-2} - \ldots + (-1)^{n-1}C_{n-1}a')$$

$$= \frac{1}{a}[(a-1)^n - (-1)^n\ {}^nC_n] = \frac{1}{a}\left\{\left(\frac{1}{3^{223}} - (-1)^n\right)\right\}$$

$$\therefore f(x) = \frac{3^{\frac{n}{223}} - (-1)^n}{\left(3^{\frac{1}{223}} + 1\right)} \Rightarrow f(2007) = \frac{3^{\frac{2007}{223}} + 1}{3^{\frac{1}{223}} + 1}$$

and $\quad f(2008) = \dfrac{3^{\frac{2008}{223}} - 1}{3^{\frac{1}{223}} + 1}$

$$\Rightarrow \quad f(2007) + f(2008) = \frac{3^{\frac{2007}{223}} + 3^{\frac{2008}{223}}}{3^{\frac{1}{223}} + 1}$$

$$= \frac{3^9 + 3^{9 + \frac{1}{223}}}{3^{\frac{1}{223}} + 1} = 3^9 \cdot \frac{\left(1 + 3^{\frac{1}{223}}\right)}{\left(1 + 3^{\frac{1}{223}}\right)} = 3^9$$

$\Rightarrow \quad 3^9 = 3^k$
Hence, $k = 9$

15. ***(5)*** We have, $\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 1 & \sin\theta & 1 \end{vmatrix}$

On applying $R_3 \to R_3 - R_1$, we get

$$\Delta = \begin{vmatrix} 1 & 3\cos\theta & 1 \\ \sin\theta & 1 & 3\cos\theta \\ 0 & \sin\theta - 3\cos\theta & 0 \end{vmatrix}$$

$$= -(\sin\theta - 3\cos\theta)\,(3\cos\theta - \sin\theta)$$

$$= (3\cos\theta - \sin\theta)^2$$

Now, $-\sqrt{9+1} \leq 3\cos\theta - \sin\theta \leq \sqrt{9+1}$

$\Rightarrow \quad 0 \leq (3\cos\theta - \sin\theta)^2 \leq 10 \quad \Rightarrow \Delta_{\max} = 10$

$\therefore \quad (\Delta_{\max})\,/\,2 = 5$

16. ***(4)*** We have, $\alpha_k = \cos\frac{k\pi}{7} + i\sin\frac{k\pi}{7} = e^{\frac{k\pi}{7}i}$

$$\therefore \quad \frac{\sum_{k=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|} = \frac{\sum_{k=1}^{12}\left|e^{\frac{(k+1)\pi}{7}i} - e^{\frac{k\pi}{7}i}\right|}{\sum_{k=1}^{3}\left|e^{\frac{(4k-1)\pi}{7}i} - e^{\frac{(4k-2)\pi}{7}i}\right|}$$

$$= \frac{\sum_{k=1}^{12}\left|e^{\frac{k\pi}{7}i}\right|\left|e^{\frac{\pi}{7}i} - 1\right|}{\sum_{k=1}^{3}\left|e^{\frac{(4k-2)\pi}{7}i}\right|\left|e^{\frac{\pi i}{7}} - 1\right|} = \frac{\sum_{k=1}^{12}1}{\sum_{k=1}^{3}1} \quad \left[\because \left|e^{\frac{k\pi}{7}i}\right| = \left|e^{\frac{(4k-2)\pi}{7}i}\right| = 1\right]$$

$$= \frac{12}{3} = 4$$

17. ***(8)*** Here, $\frac{d}{dx}\int_4^x [4t^2 - 2F'(t)]\,dt = [4x^2 - 2F'(x)]\cdot 1 - 0$

$$\Rightarrow \quad F'(x) = \frac{1}{x^2}[4x^2 - 2F'(x)] + \frac{(-2)}{x^3}\int_4^x [4t^2 - 2F'(t)]\,dt$$

$$\Rightarrow \quad F'(4) = \frac{1}{16}[64 - 2F'(4)] - \frac{1}{32}\int_4^4 [4t^2 - 2F'(t)]\,dt$$

$$\Rightarrow \quad F'(4) = \frac{1}{16}[64 - 2F'(4)]$$

$$\Rightarrow \quad \left(1 + \frac{1}{8}\right)F'(4) = 4 \Rightarrow F'(4) = \frac{32}{9}$$

Hence, $\quad \dfrac{9F'(4)}{4} = 8$

18. ***(2)*** We have, $\lim_{x \to 1}\left\{\frac{-ax + \sin(x-1) + a}{x + \sin(x-1) - 1}\right\}^{\frac{1-x}{1-\sqrt{x}}} = \frac{1}{4}$

$$\Rightarrow \quad \lim_{x \to 1}\left\{\frac{\frac{\sin(x-1)}{x-1} - a}{\frac{\sin(x-1)}{x-1} + 1}\right\}^{1+\sqrt{x}} = \frac{1}{4}$$

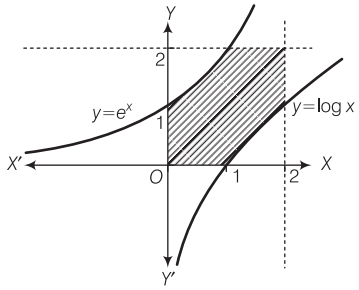
$$\Rightarrow \quad \left(\frac{1-a}{2}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \quad a = 0, a = 2$$

Paper II

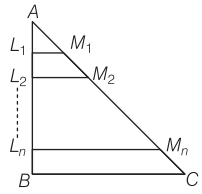
1. ***(a)*** Use $BC = I$ and $\text{tr}(kA) = k\ \text{tr}(A)$

2. ***(a)***



3. ***(b)*** Use $\tan A + \tan B + \tan C = \tan A \tan B \tan C$ and then find $\tan A, \tan B, \tan C$.

4. ***(c)***



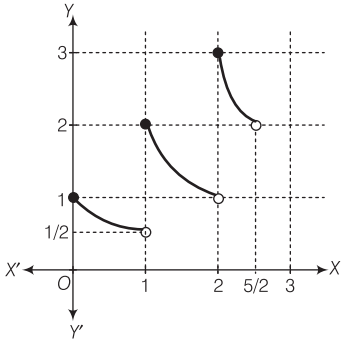
Use similarity of triangles.

5. ***(c)*** We have, $\int_0^2 f(x)\,dx = \lim\limits_{n\to\infty}\left\{\int_0^{1/2} 1\,dx + \int_{1/2}^{2/3} 1\,dx + \int_{2/3}^{3/4} 1\,dx + \ldots + \int_{\frac{n}{n-1}}^{\frac{n+1}{n}} 1\,dx\right\} + \int_1^2 dx$

$$= \left(\lim_{n\to\infty}\frac{n}{n+1}\right) + 1 = 1 + 1 = 2$$

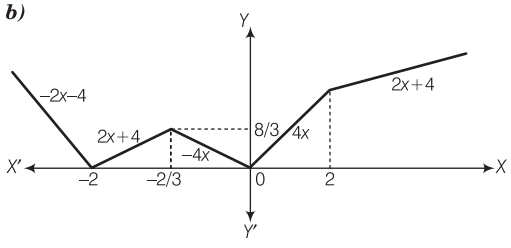
6. ***(d)*** The foot of perpendicular drawn from point $(-2, -1, 0)$ on the plane is $(0, 1, 2)$ and direction ratios of perpendicular are $2, -7, 5$.

7. ***(a, b, d)***

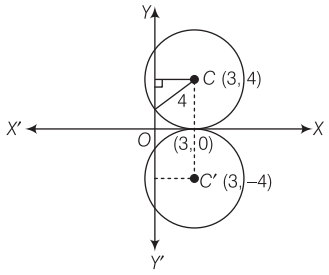


8. ***(b, c)*** Let $h(x) = f(x) - 3g(x)$ and also $h(-1) = h(0) = h(2) = 0$

9. ***(a, b)***



10. ***(a, c)***



11. ***(b, c, d)*** Here,

$$\alpha = 3\sin^{-1}\frac{6}{11} > 3\sin^{-1}\frac{6}{12} = \frac{\pi}{2}$$

and $$\beta = 3\cos^{-1}\frac{4}{9} > 3\cos^{-1}\frac{4}{8} = \pi$$

$$\therefore \quad \alpha > \frac{\pi}{2} \text{ and } \beta > \pi$$

12. ***(a, b, c, d)*** Here,

$$z = \sqrt{5 - 12i} + \sqrt{-5 - 12i}$$
$$= -1 - i, -5 + 5i, 5 - 5i, 1 + i$$

13. ***(a, b)*** $2s = a + b + c$ and $A^2 = s(s-a)(s-b)(s-c)$

$$\Rightarrow \frac{s + (s-a) + (s-b) + (s-c)}{4}$$

$$\geq \{s(s-a)(s-b)(s-c)\}^{1/4}$$

and $$\frac{(s-a) + (s-b) + (s-c)}{3} \geq \{(s-a)(s-b)(s-c)\}^{1/3}$$

14. ***(b, c)*** $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (3+2\alpha) & (3+4\alpha) & (3+6\alpha) \\ (5+2\alpha) & (5+4\alpha) & (5+6\alpha) \end{vmatrix} = -4\alpha^3$

$$\therefore \quad -4\alpha^3 = -324\alpha$$
$$\Rightarrow \quad \alpha = 0, \pm 9$$

15. ***(b)*** Equation of tangent to the hyperbola $\frac{x^2}{9} - \frac{x^2}{4} = 1$ is

$y = mx + \sqrt{9m^2 - 4}$, $m > 0$.

It is also tangent to $x^2 + y^2 - 8x = 0$.

$$\therefore \quad m = \frac{2}{\sqrt{5}}$$

$\therefore$ Tangent is $2x - \sqrt{5}y + 4 = 0$.

16. ***(a)*** Intersecting points are $A(6, 2\sqrt{3})$ and $B(6, -2\sqrt{3})$.

Sol. (Q. Nos. 17-18)

$$\because \cos A \cos B \cos C = \frac{\sqrt{3} - 1}{8}$$

and $$\sin A \sin B \sin C = \frac{3 + \sqrt{3}}{8}$$

$$\therefore \quad \tan A \tan B \tan C = \frac{3 + \sqrt{3}}{\sqrt{3} - 1}$$

$$\Rightarrow \tan A + \tan B + \tan C = \frac{3 + \sqrt{3}}{\sqrt{3} - 1}$$

Also, $\cos(A + B + C) = -1$

and $\cos A \cos B \cos C\,(1 - \Sigma \tan A \tan B) = -1$

$$\Rightarrow \quad \Sigma \tan A \tan B = 5 + 4\sqrt{3}$$

17. ***(a)*** **18.** ***(b)***

Learn Through Problems

MATRIX DETERMINANT

Section targeted to provide stepwise learning & practice through selective problems

1 The number of values of k for which the linear equations $4x + ky + 2z = 0$, $kx + 4y + z = 0$ and $2x + 2y + z = 0$ passes through a non-zero solution, is

(a) 3 (b) 2 (c) 1 (d) 0

(b) ***Step I*** Write the given set of linear equations in the form of matrix as $AX = B$.

$$\therefore \quad \begin{bmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Step II If system of equations has trivial solution, then $|A| \neq 0$ and if system of equations has non-zero solution, then $|A| = 0$.

$$\because \quad |A| = 0$$

$$\therefore \quad \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \quad k^2 - 6k + 8 = 0 \Rightarrow k = 2, 4$$

TRY THESE

1. If the trivial solution is the only solution of the system of equations $x - ky + z = 0$, $kx + 3y - kz = 0$ and $3x + y - z = 0$, then the set of values of k is

(a) $R - \{2\}$ (b) $R - \{-3\}$ (c) $\{2, -3\}$ (d) $R - \{2, -3\}$

2. Consider the system of equations in x, y and z as $x \sin 3\theta - y + z = 0$, $x \cos 2\theta + 4y + 3z = 0$ and $2x + 7y + 7z = 0$. If this system has a non-trivial solution, then for any integer n, value of θ is

(a) $\left\{n + \frac{(-1)^n}{3}\right\}\pi$ (b) $\left\{n + \frac{(-1)^n}{4}\right\}\pi$ (c) $\left\{n + \frac{(-1)^n}{6}\right\}\pi$ (d) $\frac{n\pi}{2}$

3. Consider the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$ and $a_3x + b_3y + c_3z = 0$.

If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then the system has

(a) more than two solutions
(b) one trivial and one non-trivial solutions
(c) no solution
(d) only trivial solution (0, 0, 0)

1. (d) **2.** (c) **3.** (a)

2 If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is an orthogonal matrix, then

(a) $a = 2, b = 1$ (b) $a = -2, b = -1$
(c) $a = 2, b = -1$ (d) $a = -2, b = 1$

(b) ***Step I*** Find the transpose of the given matrix A, i.e. A^T.

$$\therefore \quad A^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

Step II Find AA^T.

$$\therefore \quad AA^T = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix}$$

Step III If a matrix is orthogonal, then $AA^T = I$.

$$\therefore \quad AA^T = I$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix} 9 & 0 & a + 4 + 2b \\ 0 & 9 & 2a + 2 - 2b \\ a + 4 + 2b & 2a + 2 - 2b & a^2 + 4 + b^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step IV Equate both the matrix and find the values of unknown variables.

$\therefore \quad a + 4 + 2b = 0, 2a + 2 - 2b = 0$ and $a^2 + 4 + b^2 = 1$

$\Rightarrow a = -2$ and $b = -1$

TRY THESE

1. The matrix $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ -2 & 2 & -1 \end{bmatrix}$ is

(a) orthogonal (b) involutory (c) idempotent (d) nilpotent

2. If A is an orthogonal matrix, then A^{-1} equals

(a) A (b) A^T (c) A^2 (d) None of these

3. If $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is an orthogonal matrix, then $|abc|$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) 1

1. (a) **2.** (b) **3.** (c)

3 If A is skew-symmetric matrix of order 2 and B, C are matrices such that $B = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}$ and $C = \begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix}$ respectively, then

$A^3BC + A^5(B^2C^2) + A^7(B^3C^3) + \ldots + A^{2n+1}(B^nC^n)$ is a/an

(a) symmetric matrix (b) skew-symmetric matrix
(c) identity matrix (d) None of these

(b) ***Step I*** Find the product of BC.

$$\therefore \quad BC = \begin{bmatrix} 1 & 4 \\ 2 & 9 \end{bmatrix}\begin{bmatrix} 9 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 9-8 & -4+4 \\ 18-18 & -8+9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

$$\Rightarrow \quad BC = I \text{ or } B^n \cdot C^n = I^n = I$$

Step II Name the given matrix, say X.

$$\therefore X = A^3BC + A^5(B^2C^2) + A^7(B^3C^3) + \ldots + A^{2n+1}(B^nC^n)$$
$$= A^3I + A^5I + A^7I + \ldots + A^{2n+1}I$$
$$= A^3 + A^5 + A^7 + \ldots + A^{2n+1}$$

Step III Find X^T. If $X^T = X$, then the matrix is symmetric and if $X^T = -X$, then the matrix is skew-symmetric.

$$\therefore \quad X^T = (A^3 + A^5 + A^7 + \ldots + A^{2n+1})^T$$
$$= (A^3)^T + (A^5)^T + (A^7)^T + \ldots + (A^{2n+1})^T$$
$$= (A^T)^3 + (A^T)^5 + (A^T)^7 + \ldots + (A^T)^{2n+1}$$
$$= (-A)^3 + (-A)^5 + (-A)^7 + \ldots + (-A)^{2n+1} \quad [\because A^T = -A]$$
$$= -A^3 - A^5 - A^7 - \ldots - A^{2n+1}$$
$$= -(A^3 + A^5 + A^7 + \ldots + A^{2n+1}) = -X$$

$\therefore X$ is a skew-symmetric matrix.

TRY THESE

1. Let X and Y be two arbitrary 3×3 non-zero skew-symmetric matrices and Z be an arbitrary 3×3 non-zero symmetric matrix. Then, which of the following matrices is/are skew-symmetric?

(a) $Y^3Z^4 - Z^4Y^3$ (b) $X^{44} + Y^{44}$ (c) $X^4Z^3 - Z^3X^4$ (d) $X^{23} + Y^{23}$

2. If A is a symmetric matrix and $n \in N$, then A^n is a

(a) symmetric matrix (b) skew-symmetric matrix
(c) diagonal matrix (d) None of these

3. If A is a skew-symmetric matrix and n is odd positive integer, then A^n is a

(a) symmetric matrix (b) skew-symmetric matrix
(c) diagonal matrix (d) None of these

1. (c, d) **2.** (a) **3.** (b)

4 If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an equilateral triangle whose each side is equal to a, then $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$ is equal to

(a) $2a^2$ (b) $2a^4$
(c) $3a^2$ (d) $3a^4$

(d) ***Step I*** Find the area of Δ using determinant.

Let area of ΔABC be Δ.

$$\text{Then, } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\Rightarrow 4\Delta = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}$$

Step II Find the area of equilateral triangle whose side is a.

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4}a^2$$
$$\Rightarrow \quad 4\Delta = \sqrt{3}\,a^2$$
$$\Rightarrow \quad 16\Delta^2 = 3a^4$$
$$\therefore \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$$

TRY THESE

1. The area of triangle (in sq units) whose vertices are $(a, b+c)$, $(b, c+a)$ and $(c, a+b)$, is

(a) 0 (b) 4
(c) 8 (d) 16

2. If the area of triangle formed by the points (2, 1), (– 1, 3) and $(k, 7)$ is 7 sq units, then the value(s) of k is/are

(a) – 14 (b) 7
(c) – 7 (d) 0

1. (a) **2.** (a, d)

Mind Shaker

Romila appeared for a Maths exam. She was given 100 problems to solve. She tried to solve all of them correctly but some of them went wrong. Any how she scored 85.

Here score was calculated by subtracting two times the number of wrong answers from the number of correct answers.

Can you tell how many problems she solved correctly?

TARGET JEE 2017

TEST RIDER

Comprehensive Simulator Test Series for JEE Main & Advanced

JEE ADVANCED

(PAPER THREE SCALE UP)

Mock Questions from Complete Syllabus with Answer Key

FULL TEST 3

(COMPLETE SYLLABUS) WITH ANSWER KEY

PAPER I

SECTION 1 (MM : 18)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. The value of a so that the volume of parallelopiped formed by $\hat{i} + a\hat{j} + \hat{k}$, $\hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ become minimum, is
(a) -3 *(b)* 3 *(c)* $1/\sqrt{3}$ *(d)* $\sqrt{3}$

2. If $2^{2\pi/\sin^{-1}x} - 2(a+2)\,2^{2\pi/\sin^{-1}x} + 8a < 0$ for atleast one real x, then
(a) $\frac{1}{8} \le a < 2$ *(b)* $a < 2$
(c) $a \in R - \{2\}$ *(d)* $a \in \left[0, \frac{1}{8}\right) \cup (2, \infty)$

3. If $\sum_{i=1}^{4}(x_i^2 + y_i^2) \le 2x_1x_3 + 2x_2x_4 + 2y_2y_3 + 2y_1y_4$, then the points (x_1, y_1), (x_2, y_2), (x_3, y_3) and (x_4, y_4) are
(a) the vertices of a rectangle
(b) collinear
(c) the vertices of a trapezium
(d) None of the above

4. The eccentricity of the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ is changed at the rate of 0.1 per second. The time at which it will touch the auxiliary circle, is
(a) 2 s *(b)* 3 s *(c)* 5 s *(d)* 6 s

5. If n is selected from the set $\{1, 2, 3, \ldots, 100\}$ and the number $2^n + 3^n + 5^n$ is formed, then the total number of ways of selecting n so that the formed number is divisible by 4, is
(a) 50 *(b)* 49 *(c)* 48 *(d)* None of these

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

6. Let $f(x)$ be a real valued function defined by $f(x) = x^2 + x^2\int_{-1}^{1} t\,f(t)dt + x^3\int_{-1}^{1} f(t)dt$, then which of the following hold(s) good?

(a) $\int_{-1}^{1} t\,f(t)dt = \frac{10}{11}$ (b) $f(1) + f(-1) = \frac{30}{11}$

(c) $\int_{-1}^{1} t\,f(t)dt > \int_{-1}^{1} f(t)dt$ (d) $f(1) - f(-1) = \frac{20}{11}$

7. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then,

(a) h is increasing whenever f is increasing

(b) h is increasing whenever f is decreasing

(c) h is increasing whenever f is decreasing

(d) Nothing can be said in general

8. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$ and $S(x_4, y_4)$, then

(a) $x_1 + x_2 + x_3 + x_4 = 0$ (b) $y_1 + y_2 + y_3 + y_4 = 0$

(c) $x_1x_2x_3x_4 = c^4$ (d) $y_1y_2y_3y_4 = c^4$

9. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then the

(a) equation of ellipse is $x^2 + 2y^2 = 2$

(b) foci of ellipse are $(\pm 1, 0)$

(c) equation of ellipse is $x^2 + 2y^2 = 4$

(d) foci of ellipse are $(\pm\sqrt{2}, 0)$

10. Let $\tan x - \tan^2 x > 0$ and $|2\sin x| < 1$. Then, the intersection of which of the following two sets satisfies both the inequalities?

(a) $x > n\pi,\ n \in Z$

(b) $x > n\pi - \frac{\pi}{6},\ n \in Z$

(c) $x < n\pi - \frac{\pi}{4},\ n \in Z$

(d) $x < n\pi + \frac{\pi}{6},\ n \in Z$

11. The equation of a circle of radius 1 touching the circle $x^2 + y^2 - 2|x| = 0$, is

(a) $x^2 + y^2 + 2\sqrt{2}x + 1 = 0$

(b) $x^2 + y^2 - 2\sqrt{3}y + 2 = 0$

(c) $x^2 + y^2 + 2\sqrt{3}y + 2 = 0$

(d) $x^2 + y^2 - 2\sqrt{2}x + 1 = 0$

12. If $f(\theta) = \begin{vmatrix} \sin\theta & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & \cos\theta \\ \cos\theta & \sin\theta & \sin\theta \end{vmatrix}$, then

(a) $f(\theta) = 0$ has exactly two real solutions in $[0, \pi]$

(b) $f(\theta) = 0$ has exactly three real solutions in $[0, \pi]$

(c) range of the function $\frac{f(\theta)}{1 - \sin 2\theta}$ is $[-\sqrt{2}, \sqrt{2}]$

(d) range of the function $\frac{f(\theta)}{\sin 2\theta - 1}$ is $[-3, 3]$

13. Let $P(x)$ and $Q(x)$ be two polynomials. Suppose that $f(x) = P(x^3) + xQ(x^3)$ is divisible by $x^2 + x + 1$, then

(a) $P(x)$ is divisible by $(x-1)$ but $Q(x)$ is not divisible by $(x-1)$

(b) $Q(x)$ is divisible by $(x-1)$ but $P(x)$ is not divisible by $(x-1)$

(c) Both $P(x)$ and $Q(x)$ are divisible by $(x-1)$

(d) $f(x)$ is divisible by $(x-1)$

SECTION 3 (MM : 12)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : + 3 If only the bubble corresponding to the correct option is darkened.

Zero Marks : 0 In all other cases.

14. $\lim_{x\to\infty} \frac{\sum_{r=1}^{10}(x+r)^{2017}}{(x^{1009}+1)(2x^{1008}+1)}$ is equal to … .

15. For any real number, let $[x]$ denotes the largest integer less than or equal to x. Let f be a real valued function defined on the interval $[-10, 10]$ by

$$f(x) = \begin{cases} x - [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] - x, & \text{if } [x] \text{ is even} \end{cases}.$$

Then, the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is

16. The position vectors of the four angular points of a tetrahedron $OABC$ are $(0, 0, 0,)$, $(0, 0, 2,)$, $(0, 4, 0)$ and $(6, 0, 0)$, respectively. A point P inside the tetrahedron is at the same distance r from the four plane faces of the tetrahedron. Find the value of $9r$.

17. If a tangent of slope 2 for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then the maximum value of ab is … .

18. The number of points of non-differentiability of the function $f(x) = \max\{\sin^{-1}|\sin x|, \cos^{-1}|\sin x|\}$, $0 < x < 2\pi$, is … .

PAPER II

SECTION 1 (MM : 18)

- This section contains **SIX** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −1 In all other cases.

1. If the vertices of a variable triangle are $(3, 4)$, $(5\cos\theta, 5\sin\theta)$ and $(5\sin\theta, -5\cos\theta)$, where $\theta \in R$, then the locus of its orthocentre is
(a) $(x + y - 1)^2 + (x - y - 7)^2 = 100$
(b) $(x + y - 7)^2 + (x - y - 1)^2 = 100$
(c) $(x + y - 7)^2 + (x + y - 1)^2 = 100$
(d) $(x + y - 7)^2 + (x - y + 1)^2 = 100$

2. $\lim\limits_{n \to \infty} (1 + 3^{-1})(1 + 3^{-2})(1 + 3^{-4})(1 + 3^{-8}) \ldots (1 + 3^{-2^n})$ is equal to
(a) 1 *(b)* $\frac{1}{2}$ *(c)* $\frac{3}{2}$ *(d)* None of these

3. Let a_n be the nth term of an AP. If $\sum\limits_{r=1}^{10^{99}} a_{2r} = 10^{100}$ and $\sum\limits_{r=1}^{10^{99}} a_{2r-1} = 10^{99}$, then the common difference of the AP is
(a) 1 *(b)* 9
(c) 10 *(d)* 10^{99}

4. The maximum area of the triangle formed by the complex coordinates z, z_1 and z_2 which satisfy the relations $|z - z_1| = |z - z_2|$, $\left|z - \left(\frac{z_1 + z_2}{2}\right)\right| \le r$, where $r > |z_1 - z_2|$, is
(a) $\frac{1}{2}|z_1 - z_2|^2$ *(b)* $\frac{1}{2}|z_1 - z_2|r$
(c) $\frac{1}{2}|z_1 - z_2|^2 r^2$ *(d)* $\frac{1}{2}|z_1 - z_2|r^2$

5. The total number of functions, $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ that are onto and $f(i) \ne i$, is
(a) 9 *(b)* 44
(c) 16 *(d)* None of these

6. The number of ordered 5-tuple (u, v, w, x, y), where $u, v, w, x, y \in [1, 11]$, which satisfy the inequality $2^{\sin^2 u + 3\cos^2 v} \cdot 3^{\sin^2 w + \cos^2 x} \cdot 5^{\cos^2 y} \ge 720$, is
(a) 432 *(b)* 216
(c) 243 *(d)* None of these

SECTION 2 (MM : 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +4 If only the bubble(s) corresponding to all correct option(s) is(are) darkened.
Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided no incorrect option is darkened.
Zero Marks : 0 If none of the bubbles is darkened.
Negative Marks : −2 In all other cases.

For example, if (a), (c) and (d) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (a) and (d) will result in +2 marks and darkening (a) and (b) will result in −2 marks, as a wrong option is also darkened.

7. If $(a, 0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ has
(a) exactly one real root in $(-1, 0]$
(b) exactly one real root in $[2, 5]$
(c) distinct roots greater than -1
(d) distinct roots less than 5

8. If $f(x) = \begin{vmatrix} (1+x)^a & (1+2x)^b & 1 \\ 1 & (1+x)^a & (1+2x)^b \\ (1+2x)^b & 1 & (1+x)^a \end{vmatrix}$, then
(a) constant term of $f(x) = 0$
(b) the coefficient of x in $f(x)$ is zero
(c) constant term is $a + b$
(d) None of the above

9. If $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$ with $p \neq 0$, then
(a) $\sin^2(\alpha+\beta) + p\sin(\alpha+\beta)\cos(\alpha+\beta) + q\cos^2(\alpha+\beta) = q$
(b) $\tan(\alpha+\beta) = \dfrac{p}{q-1}$
(c) $\cos(\alpha+\beta) = -p$
(d) $\sin(\alpha+\beta) = 1-q$

10. Suppose $x_1, x_2, \ldots, x_n (n > 2)$ are real numbers such that $x_i = -x_{n-i+1}$ for $1 \le i \le n$. Consider the sum $S = \sum\sum\sum x_i x_j x_k$, where $1 \le i, j, k \le n$ and i, j, k are distinct, then which of the following is/are not true?
(a) $S = n! \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4 \ldots x_n$ *(b)* $S = (n-3)(n-4)$
(c) $S = (n-3)(n-4)(n-5)$ *(d)* $S = 0$ for all n

11. Identify the pair(s) of functions which are identical.
(a) $y = \tan(\cos^{-1}x)$; $y = \dfrac{\sqrt{1-x^2}}{x}$
(b) $y = \tan(\cot^{-1}x)$; $y = \dfrac{1}{x}$
(c) $y = \sin(\arctan x)$; $y = \dfrac{x}{\sqrt{1+x^2}}$
(d) $y = \cos(\arctan x)$; $y = \sin(\operatorname{arccot} x)$

12. In ΔABC with fixed base BC, the vertex A moves such that $\cos B + \cos C = 4\sin^2\dfrac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then
(a) $b + c = 4a$
(b) $b + c = 2a$
(c) locus of point A is an ellipse
(d) locus of point A is a pair of straight lines

13. If A_i is the area bounded by $|x - a_i| + |y| = b_i$, where $a_{i+1} = a_i + \dfrac{3}{2}b_i$ and $b_{i+1} = \dfrac{b_i}{2}$; $a_1 = 0$, $b_1 = 32$, then
(a) $A_3 = 128$ *(b)* $A_3 = 256$
(c) $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} A_i = \dfrac{8}{3}(32)^2$ *(d)* $\lim\limits_{n\to\infty}\sum\limits_{i=1}^{n} A_i = \dfrac{4}{3}(16)^2$

14. If E_1 and E_2 are two events such that $P(E_1) = \dfrac{1}{4}$, $P(E_2 / E_1) = \dfrac{1}{2}$ and $P(E_1 / E_2) = \dfrac{1}{4}$, then
(a) E_1 and E_2 are independent
(b) E_1 and E_2 are exhaustive
(c) E_2 is twice as likely to occur as E_1
(d) probability of the events $E_1 \cap E_2$, E_1 and E_2 are in GP

SECTION 3 (MM : 12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (a), (b), (c) and (d). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	:	+ 3 If only the bubble corresponding to the correct option is darkened.
Zero Marks	:	0 In all other cases.

Passage 1 (Q. Nos. 15-16) $f(x)$ is a continuous function satisfying $x^4 - 4x^2 \le f(x) \le 2x^2 - x^3$ for all $x \in [0, 2]$, such that the area bounded by $y = f(x)$, $y = x^4 - 4x^2$, the Y-axis and the line $x = t\,(0 \le t \le 2)$ is k times the area bounded by $y = f(x)$, $y = 2x^2 - x^3$, Y-axis and the line $x = t\,(0 \le t \le 2)$.

15. The value of $\int_{-1}^{1} f(x)\,dx$ is
(a) $\dfrac{2}{15(k+1)}(23 - 10k)$ *(b)* $\dfrac{2}{15(k+1)}(23 + 10k)$
(c) $\dfrac{2}{15(k+1)}(10k - 17)$ *(d)* $\dfrac{2}{15(k+1)}(10k + 17)$

16. If $k = 0$, then $f(x)$ attains local maxima at
(a) 0 *(b)* $\sqrt{2}$
(c) $-\sqrt{2}$ *(d)* None of these

Passage 2 (Q. Nos. 17-18) An equation $a_0 + a_1x + a_2x^2 + \ldots + a_{99}x^{99} + x^{100} = 0$ has roots ${}^{99}C_0, {}^{99}C_1, {}^{99}C_2, \ldots, {}^{99}C_{99}$.

17. The value of a_{99} is
(a) 2^{98} *(b)* 2^{99}
(c) -2^{99} *(d)* None of these

18. The value of a_{98} is
(a) $\dfrac{2^{198} - {}^{198}C_{99}}{2}$ *(b)* $\dfrac{2^{198} + {}^{198}C_{99}}{2}$

Answers

Paper I

1. (c)	**2.** (d)	**3.** (a)	**4.** (c)	**5.** (b)	**6.** (b,d)	**7.** (a,c)	**8.** (a,b,c,d)	**9.** (a,b)	**10.** (a,d)
11. (b,c)	**12.** (a,c)	**13.** (c,d)	**14.** (5)	**15.** (4)	**16.** (6)	**17.** (4)	**18.** (7)		

Paper II

1. (d)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (a,b,c,d)	**8.** (a,b)	**9.** (a,b)	**10.** (a,b,c)
11. (a,b,c,d)	**12.** (b,c)	**13.** (a,c)	**14.** (a,c,d)	**15.** (c)	**16.** (a)	**17.** (c)	**18.** (a)		

Er. Love Agrawal

JEE ADVANCED DRILL

A collection of 30 innovative problems on JEE Advanced pattern from complete syllabus

Single Option Correct Type

1. If $ax_1^2 + by_1^2 + cz_1^2 = ax_2^2 + by_2^2 + cz_2^2$ $= ax_3^2 + by_3^2 + cz_3^2 = d$ and $ax_2x_3 + by_2y_3 + cz_2z_3$ $= ax_3x_1 + by_3y_1 + cz_3z_1 = ax_1x_2 + by_1y_2 + cz_1z_2 = f$, then $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ is equal to

(a) $(f-d)\left(\frac{d+2f}{abc}\right)^{1/2}$ (b) $(d-f)\left(\frac{d+2f}{abc}\right)^{1/2}$

(c) $(d-f)\left(\frac{f+2d}{abc}\right)^{1/2}$ (d) $(f-d)\left(\frac{f+2d}{abc}\right)^{1/2}$

2. If z_1 and z_2 are two complex numbers which satisfy the relation $z + \bar{z} = 2|z-1|$ and $\arg(z_1 - z_2) = \frac{\pi}{4}$, then $\text{Im}(z_1 + z_2)$ equals

(a) 2 (b) 3 (c) 4 (d) 1

3. If $g(x) = \sin^{-1}\{4-(x-7)^3\}^{1/5}$, then its inverse is

(a) $7 + (4 + \sin^5 x)^{1/3}$ (b) $7 - (4 - \sin^5 x)^{1/3}$

(c) $7 + (4 - \sin^5 x)^{1/3}$ (d) $7 + (-4 + \sin^5 x)^{1/3}$

4. If f is a function such that $f(xy) = f(x)\cdot f(y), \forall\, x, y \in R$ and $f(1+x) = 1 + x\{1 + g(x)\}$, where $\lim_{x\to 0} g(x) = 0$, , then the value of $\int_1^2 \frac{f(x)}{f'(x)}\cdot\frac{1}{1+x^2}\,dx$ is

(a) $\frac{1}{2}\log\left(\frac{2}{5}\right)$ (b) $\frac{1}{2}\log\left(\frac{5}{2}\right)$

(c) $\frac{3}{2}\log\left(\frac{5}{2}\right)$ (d) $\frac{3}{2}\log\left(\frac{2}{5}\right)$

5. If $a_0, a_1, a_2, \ldots, a_{2n}$ are the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending power of x, then $a_0^2 - a_1^2 + a_2^2 - \ldots + a_{2n}^2$ is equal to

(a) $-a_n$ (b) $2a_n$ (c) a_n (d) $-2a_n$

More than One Option Correct Type

6. If the normal $y = mx - 2am - am^3$ to the parabola $y^2 = 4ax$ subtends a right angle at the vertex, then m equals

(a) $-\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $-\sqrt{3}$

7. If $f(x_1 - x_2)$, $f(x_1)\,f(x_2)$ and $f(x_1 + x_2)$ are in AP for all x_1, x_2 and $f(0) \neq 0$, then

(a) $f'(5) = f(-5)$ (b) $f(5) = f(-5)$

(c) $f'(5) - f'(-5) = 0$ (d) $f'(5) + f'(-5) = 0$

8. If a function $f(x)$ satisfying the differential equation $\frac{dy}{dx} - y\cot x + \frac{\sin x}{x^2} = 0$ is such that $y \to 0$ as $x \to \infty$, then

(a) $\int_0^{\pi/2} f(x)\,dx > 1$ (b) $f(x)$ is an even function

(c) $\lim_{x\to 0} f(x) = 1$ (d) $\int_0^{\pi/2} f(x)\,dx < \frac{\pi}{2}$

9. If $4\log_2^2 x + 1 = 2\log_2 y$ and $\log_2 x^2 \geq \log_2 y$, then

(a) $x = \sqrt{2}$ (b) $y = \sqrt{3}$ (c) $y = 2$ (d) $x^2 = 2$

10. If one root of the equation $e^a x^2 - e^{2a}x + e^a - 1 = 0$ lies between 1 and 2, then which of the statements is/are correct?

(a) $2e^{2a} - 5e^a + 1 < 0$ (b) $e^{2a} - 5a^a + 1 < 0$

(c) $\log\left(\frac{5-\sqrt{17}}{4}\right) < a < \log\left(\frac{5+\sqrt{17}}{4}\right)$ (d) $(e^{2a} - 1)^2 > 0$

11. If the sides a, b and c of ΔABC is such that $\frac{a}{1+\alpha^2\beta^2} = \frac{b}{\alpha^2+\beta^2} = \frac{c}{(1-\alpha^2)(1+\beta^2)}$, then

(a) $A = 2\tan^{-1}(\alpha/\beta)$ (b) $B = 2\tan^{-1}\alpha\beta$

(c) area of $\Delta ABC = \frac{\alpha\beta ab}{\alpha^2+\beta^2}$ (d) area of $\Delta ABC = \frac{\alpha\beta bc}{\alpha^2+\beta^2}$

12. If the unit vectors $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are perpendicular and the unit vector $\hat{\mathbf{c}}$ is inclined at an angle θ to both $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$. Let $\hat{\mathbf{c}} = \alpha\hat{\mathbf{a}} + \beta\hat{\mathbf{b}} + \gamma(\hat{\mathbf{a}} \times \hat{\mathbf{b}})$, then

(a) $\gamma^2 = -\cos 2\theta$ (b) $\beta^2 = \frac{1+\cos 2\theta}{2}$

(c) $\gamma^2 = 1 - 2\alpha^2$ (d) $\alpha = \beta$

13. Let $f(x) = x^3 + e^{x/2}$ and $g(x) = f^{-1}(x)$. Then,

(a) $g'(1) = 2$ (b) $g(1) = 0$

(c) $f'(0) = \frac{1}{2}$ (d) $f(0) = 3$

Single Integer Type

14. If $\tan(2\pi|\sin\theta|) = \cot(2\pi|\cos\theta|)$, where $\theta \in R$ and $f(x) = (|\sin\theta| + |\cos\theta|)^x$, then $\lim_{x\to\infty}\left[\frac{2}{f(x)}\right]$ is equal to ....., where $[\cdot]$ represents greatest integer function.

15. The area of a triangle formed by the line $x + y = 3$ and angle bisector of the pairs of straight lines $x^2 - y^2 + 2y = 1$ is ..... .

16. In a multiple choice question, there are five alternative answers, of which one or more than one option is correct. A candidate will get marks on the question, if he ticks all the correct answers. If he decides to tick answers at random, then the chances should be allowed so that the probability of his getting marks on the question exceeds $\frac{1}{8}$, is ..... .

17. If the area between the curve $y = 2x^4 - x^2$, the X-axis and the ordinate of two minima of curve, is $\frac{k}{720}$ sq unit, then the value of k is ..... .

18. The sum of the terms of GP, $a + ar + ar^2 + \ldots\infty$, where a is the value of x for which the function $7 + 2x\log_e 25 - 5^{x-1} - 5^{2-x}$ has the greatest value and r is $\lim_{x\to 0}\int_0^x \frac{t^2}{\{x^2\tan(\pi + x)\}}dt$, is ..... .

19. The value of $p(p > 0)$ such that the length of the longest interval in which the function $f(x) = \sin^{-1}|\sin px| + \cos^{-1}(\cos px)$ is constant is $\frac{\pi}{4}$, is ..... .

20. A man has 8 relatives, 5 of them are ladies and 3 gentlemen, while his wife has also 8 relatives, 3 of them are ladies and 5 gentlemen. If number of ways can they invite a dinner party of 4 ladies and 4 gentlemen so that there are four of them man's relatives four of the wife's relatives is λ, then $(\lambda - 1845)$ is ..... .

21. If the numbers $3^{2\sin 2x - 1}$, 14 and $3^{4 - 2\sin 2x}$ form first three terms of an AP and if its 10th term is equal to $59k$, then k is equal to ..... .

Comprehensive Based Type

Passage 1 (Q. Nos 22-23) C_1 is a circle having centre $(0, a)$ and radius $b(b < a)$. A variable circle centred at $(c, 0)$ with radius r and touching the circle C_1 meets the X-axis at B and C and $P(0, p)$ is a point on the Y-axis.

22. $\angle BPC$ is equal to

(a) $\left|\frac{2pr}{c^2 - r^2 + p^2}\right|$ (b) $\left|\frac{pr}{c^2 - r^2 + p^2}\right|$

(c) $\left|\frac{p}{c^2 - r^2 + p^2}\right|$ (d) $\left|\frac{r}{c^2 - r^2 + p^2}\right|$

23. If $\angle BPC$ is a constant for any choice of c, then the coordinates of P are

(a) $(0, \pm\sqrt{a^2 + b^2})$ (b) $(0, \pm\sqrt{a^2 - b^2})$

(c) $(\pm\sqrt{a^2 + b^2}, 0)$ (d) $(\pm\sqrt{a^2 - b^2}, 0)$

Passage 2 (Q. Nos 24-25) Consider the lines

$L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$ and $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$.

24. The unit vector perpendicular to both L_1 and L_2 is

(a) $\frac{-\hat{\mathbf{i}} + 7\hat{\mathbf{j}} + 7\hat{\mathbf{k}}}{\sqrt{99}}$ (b) $\frac{7\hat{\mathbf{i}} - 7\hat{\mathbf{j}} - \hat{\mathbf{k}}}{\sqrt{99}}$

(c) $\frac{-\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}}}{5\sqrt{3}}$ (d) $\frac{\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 5\hat{\mathbf{k}}}{5\sqrt{3}}$

25. The distance of the point (1, 1, 1) from the plane passing through the point $(-1, -2, -1)$ whose normal is perpendicular to both the lines L_1 and L_2, is

(a) $\frac{13}{\sqrt{75}}$ units (b) $\frac{2}{\sqrt{75}}$ units

(c) $\frac{23}{\sqrt{75}}$ units (d) $\frac{7}{\sqrt{75}}$ units

Passage 3 (Q. Nos. 26-27) If $f(x)$ is a differential of real valued function satisfying $f'(x) + 2f(x) \le 1$, then it can be adjusted as $e^{2x}f'(x) + 2e^{2x}f(x) \le e^{2x}$ or

$$\frac{d}{dx}\left\{e^{2x}f(x) - \frac{e^{2x}}{2}\right\} \le 0 \text{ or } \frac{d}{dx}\left\{e^{2x}\left(f(x) - \frac{1}{2}\right)\right\} \le 0.$$

Here, e^{2x} is called integrating factor which helps in creating single differential coefficient as shown above.

26. If $P(1) = 0$ and $\frac{dP(x)}{dx} > P(x)$ for all $x \geq 1$, then

(a) $P(x) > 0, \forall\, x > 1$
(b) $P(x)$ is constant function
(c) $P(x)$ is a decreasing function
(d) $P(x)$ is an increasing function

27. If $\phi(x_0) = 0$ for some $x = x_0$ and $\frac{d}{dx}\phi(x) > 2ax\,\phi(x)$ for all $x \geq x_0$, where $a > 0$, then

(a) $\phi(x) = 0$ has root for $x > x_0$
(b) $\phi(x) = 0$ has no root for $x > x_0$
(c) $\phi(x)$ is a constant function
(d) $\phi(x)$ is a decreasing function

Matching Type

28. Match the following columns.

	Column I		Column II
A.	If $\lim_{n\to\infty} n\int_0^{\pi/2}(1 - \sqrt[n]{\sin x})dx = \frac{k\pi}{2}\log 2$, then k equals	p.	1
B.	If $f(x) = \int_0^{g(x)} \frac{dt}{\sqrt{1+t^3}}$, where $g(x) = \int_0^{\cos x}(1+\sin t^2)dt$, then the value of $2f'\left(\frac{\pi}{2}\right)$ is	q.	$e^{-1/4}$
C.	$\lim_{n\to\infty}\left(\frac{1^2\cdot 2^2\cdot 3^3\ldots(n-1)^{n-1}\cdot n^n}{n^{1+2+3+\cdots+n}}\right)$ is equal to	r.	-1
D.	If $\int_0^{\pi}\frac{x\,\lvert\sin x\rvert}{1+\lvert\cos x\rvert}dx = b\pi\log 2$, then b is equal to	s.	-2

29. Match the following columns.

	Column I		Column II
A.	If A and B are square matrices and A is non-singular matrix, then $(A^{-1}BA)^n$, $n \in I^+$, is equal to	p.	$A*B + A*C$
B.	If A, B and C are 2×2 matrices with entries from the set of real numbers such that $A*B = \frac{1}{2}(AB' + A'B)$, then $A*(B+C)$ is equal to	q.	Zero matrix
C.	If matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for $n \geq 2$, A^n is equal to	r.	$A^{-1}B^nA$
D.	If $A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, then $\lim_{n\to\infty}\frac{A^{-n}}{n^2}$ is equal to	s.	$nA - (n-1)I$

30. Match the following columns.

	Column I		Column II
A.	The distance between foci of the curve represented by the equation $x = 1 + 4\cos\theta$ and $y = 2 + 3\sin\theta$, is	p.	2
B.	If e_1 and e_2 are the eccentricities of hyperbola and conjugate hyperbola and also e_1 and e_2 are the roots of the equation $x^2 - ax + 2 = 0$, then the value of a is	q.	-1
C.	If the normal to the rectangular hyperbola $xy = c^2$ at the point (c, c) on it intersects it at $(ct', c/t')$, then t' is equal to	r.	$2\sqrt{7}$
D.	If P and Q are the points $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ and normals at P and Q meet on the parabola $y^2 = 4ax$, then t_1t_2 is equal to	s.	$\pm 2\sqrt{2}$

Mind Twister

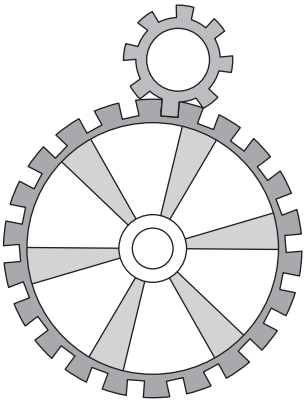
Solve A Dilemma

What is wrong with this proof?

$$a = b$$
$$a^2 = ab$$
$$a^2 - b^2 = ab - b^2$$
$$(a-b)(a+b) = b(a-b)$$
$$a + b = b$$
$$2b = b$$
$$2 = 1$$

Mind Twister

Here is a cog-wheel that has eight teeth. It is coupled with a cog-wheel of 24 teeth. Can you tell how many times the small cog-wheel must rotate on its axis to circle around the big?



Answers with Explanation

1. ***(b)*** Let $\Delta=\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}$

$$\therefore \quad \Delta^2=\frac{1}{abc}\begin{vmatrix}x_1 & y_1 & z_1\\ x_2 & y_2 & z_2\\ x_3 & y_3 & z_3\end{vmatrix}\begin{vmatrix}ax_1 & by_1 & cz_1\\ ax_2 & by_2 & cz_2\\ ax_3 & by_3 & cz_3\end{vmatrix}$$

On multiplying row by row and using the given relations, we get

$$\Delta^2=\frac{1}{abc}\begin{vmatrix}d & f & f\\ f & d & f\\ f & f & d\end{vmatrix}$$

$$\Rightarrow \quad \Delta^2=\frac{1}{abc}\begin{vmatrix}d+2f & f & f\\ d+2f & d & f\\ d+2f & f & d\end{vmatrix}\quad[\because C_1\to C_1+C_2+C_3]$$

$$\Rightarrow \quad \Delta^2=\frac{(d+2f)}{abc}\begin{vmatrix}1 & f & f\\ 1 & d & f\\ 1 & f & d\end{vmatrix}$$

$$\Rightarrow \quad \Delta^2=\frac{d+2f}{abc}\begin{vmatrix}1 & f & f\\ 0 & d-f & 0\\ 1 & f & d\end{vmatrix}\quad[\because R_2\to R_2-R_1]$$

$$\Rightarrow \quad \Delta^2=\frac{(d+2f)(d-f)^2}{abc}$$

$$\therefore \quad \Delta=(d-f)\left\{\frac{d+2f}{abc}\right\}^{1/2}$$

2. ***(a)*** We have, $z+\bar{z}=2|z-1|$ and $\arg(z_1-z_2)=\frac{\pi}{4}$

Put $z=x+iy$

$$\Rightarrow \quad (x+iy)+(x-iy)=2\sqrt{(x-1)^2+y^2}$$

$$\Rightarrow \quad x^2=(x-1)^2+y^2$$

$$\Rightarrow \quad y^2-2x+1=0 \qquad \ldots(i)$$

$\therefore$ Roots are (x_1,y_1) and (x_2,y_2).

Since, $\arg(z_1-z_2)=\frac{\pi}{4}$

$$\therefore \quad \tan^{-1}\left(\frac{y_1-y_2}{x_1-x_2}\right)=\frac{\pi}{4}$$

$$\Rightarrow \quad y_1-y_2=(x_1-x_2) \qquad \ldots(ii)$$

Using Eq. (i), $y_1^2-2x_1+1=0$ and $y_2^2-2x_2+1=0$

$$\Rightarrow \quad y_1^2-y_2^2=2(x_1-x_2)$$

$$\Rightarrow \quad y_1+y_2=2$$

$$\Rightarrow \quad \mathrm{Im}(z_1)+\mathrm{Im}(z_2)=2$$

$$\therefore \quad \mathrm{Im}(z_1+z_2)=2$$

3. ***(c)*** Let $y=g(x)=\sin^{-1}\{4-(x-7)^3\}^{1/5}$

$$\Rightarrow \quad (\sin y)^5=4-(x-7)^3$$

$$\Rightarrow \quad (x-7)=(4-\sin^5 y)^{1/3}$$

$$\Rightarrow \quad x=7+(4-\sin^5 y)^{1/3}$$

$$\Rightarrow \quad g^{-1}(y)=7+(4-\sin^5 y)^{1/3}$$

$$\therefore \quad g^{-1}(x)=7+(4-\sin^5 x)^{1/3}$$

4. ***(b)*** Here, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f\left\{x\left(1+\frac{h}{x}\right)\right\}-f(x)}{h}$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f(x)\cdot f\left(1+\frac{h}{x}\right)-f(x)}{h}$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f(x)\left\{1+\frac{h}{x}\left[1+g\left(\frac{h}{x}\right)\right]\right\}-f(x)}{h}$$

$$\Rightarrow \quad f'(x)=\lim_{h\to 0}\frac{f(x)\left\{1+\frac{h}{x}\left\{1+g\left(\frac{h}{x}\right)\right\}-1\right\}}{h}$$

$$\Rightarrow \quad f'(x)=\frac{f(x)}{x} \qquad [\because \lim_{h\to 0} g(x)=0]$$

$$\therefore \int_1^2 \frac{f(x)}{f'(x)}\cdot\frac{1}{1+x^2}dx=\int_1^2\frac{x}{1+x^2}dx$$

$$=\frac{1}{2}[\log(1+x^2)]_1^2$$

$$=\frac{1}{2}\log\left(\frac{5}{2}\right)$$

5. ***(c)*** We have,

$$(1+x+x^2)^n=a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n} \qquad \ldots(i)$$

Replace x by $-\frac{1}{x}$ in Eq. (i), we get

$$\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n=a_0-\frac{a_1}{x}+\frac{a_2}{x^2}-\ldots+\frac{a_{2n}}{x^{2n}} \qquad \ldots(ii)$$

On multiplying Eqs. (i) and (ii), we get

$$(1+x+x^2)^n\times\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n$$

$$=(a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n})$$

$$\times\left(a_0-\frac{a_1}{x}+\frac{a_2}{x^2}-\ldots+\frac{a_{2n}}{x^{2n}}\right)$$

$$\Rightarrow \quad \frac{(1+x^2+x^4)^n}{x^{2n}}=(a_0+a_1x+a_2x^2+\ldots+a_{2n}x^{2n})$$

$$\times\left(a_0-\frac{a_1}{x}+\frac{a_2}{x^2}-\ldots+\frac{a_{2n}}{x^{2n}}\right) \qquad \ldots(iii)$$

Constant term in RHS $=a_0^2-a_1^2+a_2^2-\ldots+a_{2n}^2$

Now, constant term in $\frac{(1+x^2+x^4)^n}{x^{2n}}$

$=$ Coefficient of x^{2n} in $(1+x^2+x^4)^n$

$=(a_n)$ [replace x by x^2 in Eq. (i)]

But Eq. (iii) is an identity.

$\therefore$ Constant term in RHS = Constant term in LHS

$\Rightarrow a_0^2-a_1^2+a_2^2-\ldots+a_{2n}^2=a_n$

6. ***(a, c)*** We have, $y = mx - 2am - am^3$

$$\Rightarrow \quad \frac{mx - y}{2am + am^3} = 1 \qquad \ldots(i)$$

and $y^2 = 4ax \Rightarrow y^2 = 4ax \cdot 1$

$$\Rightarrow \quad y^2 = 4ax\left(\frac{mx - y}{2am + am^3}\right) \qquad [\text{from Eq. (i)}]$$

$$\Rightarrow \quad y^2 = \frac{4x(mx - y)}{2m + m^3}$$

$$\Rightarrow \quad (2m + m^3)y^2 - 4mx^2 + 4xy = 0 \qquad \ldots(ii)$$

$\because$ Angle between the lines represented by Eq. (ii) is $\frac{\pi}{2}$.

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$$\Rightarrow \quad 2m + m^3 - 4m = 0 \Rightarrow m^3 - 2m = 0$$

$$\Rightarrow \quad m^2 = 2,\ m \neq 0$$

$$\Rightarrow \quad m = \pm\sqrt{2}$$

7. ***(b, d)*** Since, $f(x_1 - x_2), f(x_1)\,f(x_2)$ and $f(x_1 + x_2)$ are in AP.

$$\Rightarrow f(x_1 + x_2) + f(x_1 - x_2) = 2f(x_1)\,f(x_2), \forall\, x_1, x_2 \qquad \ldots(i)$$

On putting $x_1 = 0, x_2 = 0$ in Eq. (i), we get

$$f(0) + f(0) = 2f(0)\,f(0)$$

$$\Rightarrow \quad f(0) = 1 \qquad [\because f(0) \neq 0]$$

On putting $x_1 = 0, x_2 = x_1$ in Eq. (i), we get

$$f(x_1) + f(-x_1) = 2f(0)\,f(x_1)$$

$$\Rightarrow \quad f(x_1) = f(-x_1) \qquad \ldots(ii)$$

$$\Rightarrow \quad f(5) = f(-5)$$

On differentiating Eq. (ii) w.r.t. x_1, we get

$$f'(x_1) + f'(-x_1) = 0 \Rightarrow f'(5) + f'(-5) = 0$$

8. ***(a, b, c, d)*** Given, $\frac{dy}{dx} - y\cot x + \frac{\sin x}{x^2} = 0$

$$\Rightarrow \frac{dy}{dx} - y\cot x = \frac{-\sin x}{x^2}$$

Now $\quad \text{IF} = e^{\int -\cot x\,dx} = e^{-\log \sin x} = \frac{1}{\sin x}$

$\therefore$ Required solution is given by

$$y \cdot \frac{1}{\sin x} = \int\left(\frac{-\sin x}{x^2}\right)\cdot\left(\frac{1}{\sin x}\right)dx + C \Rightarrow y \cdot \frac{1}{\sin x} = \frac{1}{x} + C$$

As $x \to \infty$, $y \to 0$ then $C \to 0$

$$\therefore \quad y = \frac{\sin x}{x} \Rightarrow \lim_{x \to 0} f(x) = 1 \qquad \ldots(i)$$

Now, $\quad I = \int_0^{\pi/2} \frac{\sin x}{x}\,dx$

Since, $\frac{\sin x}{x}$ is decreasing.

$$\therefore \quad x > 0 \Rightarrow f(x) < f(0)$$

$$\Rightarrow \quad \int_0^{\pi/2} f(x)\,dx < \frac{\pi}{2}$$

and $\quad x < \frac{\pi}{2} \Rightarrow f(x) > f\left(\frac{\pi}{2}\right) \Rightarrow \int_0^{\pi/2} f(x)\,dx > 1$

9. ***(a, c, d)*** Given, $4\log_2^2 x + 1 = 2\log_2 y \qquad \ldots(i)$

and $\quad \log_2 x^2 \geq \log_2 y \qquad \ldots(ii)$

On substituting $\log_2 y$ from Eq. (i) in Eq. (ii), we get

$$2\log_2 x \geq \frac{1}{2}(4\log_2^2 x + 1)$$

$$\Rightarrow \quad 4\log_2 x \geq 4(\log_2 x)^2 + 1$$

$$\Rightarrow \quad 4(\log_2 x)^2 - 4\log_2 x + 1 \leq 0$$

$$\Rightarrow \quad (2\log_2 x - 1)^2 \leq 0$$

$$\Rightarrow \quad 2\log_2 x - 1 = 0 \Rightarrow x = \sqrt{2}$$

and $\quad y = 2 \qquad [\text{from Eq. (i)}]$

10. ***(a, c, d)*** Let $f(x) = e^a x^2 - e^{2a}x + e^a - 1$ represents a parabola opening upwards.

Also, discriminant of $f(x) = 0$ is

$$D = e^{4a} - 4e^a(e^a - 1)$$

$$\Rightarrow \quad (e^{2a} - 2)^2 + 4(e^a - 1) > 0 \Rightarrow a > 0$$

So, the roots are real and distinct for $a > 0$.

$\Rightarrow$ Exactly one root will lie between 1 and 2, if $f(1)\,f(2) < 0$.

$$\Rightarrow (e^a - e^{2a} + e^a - 1)(4e^a - 2e^{2a} + e^a - 1) < 0$$

$$\Rightarrow \quad 2e^{2a} - 5e^a + 1 < 0$$

$$\Rightarrow \quad \frac{5 - \sqrt{17}}{4} < e^a < \frac{5 + \sqrt{17}}{4}$$

$$\Rightarrow \quad \log\left(\frac{5 - \sqrt{17}}{4}\right) < a < \log\left(\frac{5 + \sqrt{17}}{4}\right)$$

11. ***(a, b, d)*** Given,

$$\frac{a}{1 + \alpha^2\beta^2} = \frac{b}{\alpha^2 + \beta^2} = \frac{c}{(1 - \alpha^2)(1 + \beta^2)} = \frac{2s}{2 + 2\beta^2}$$

$$\Rightarrow \quad \frac{s}{1 + \beta^2} = \frac{a}{1 + \alpha^2\beta^2} \Rightarrow \frac{s - a}{\beta^2(1 - \alpha^2)} = \frac{a}{1 + \alpha^2\beta^2}$$

$$\Rightarrow \quad s - a = \frac{a(1 - \alpha^2)\beta^2}{(1 + \alpha^2\beta^2)}$$

Similarly, $s - b = \frac{b(1 - \alpha^2)}{\alpha^2 + \beta^2}$

$$\therefore \quad s - c = \frac{c\alpha^2}{1 - \alpha^2} \text{ and } s = \frac{b(1 + \beta^2)}{\alpha^2 + \beta^2}$$

$$\therefore \quad \tan^2(A/2) = \frac{(s - b)(s - c)}{s(s - a)} = \frac{\frac{b(1 - \alpha^2)}{\alpha^2 + \beta^2}\cdot\frac{c\alpha^2}{1 - \alpha^2}}{\frac{b(1 + \beta^2)}{\alpha^2 + \beta^2}\cdot\frac{a(1 - \alpha^2)\beta^2}{1 + \alpha^2\beta^2}}$$

$$\left[\because a = \frac{c(1 + \alpha^2\beta^2)}{(1 - \alpha^2)(1 + \beta^2)}\right]$$

$$= \frac{\alpha^2}{\beta^2}$$

$$\therefore \quad A = 2\tan^{-1}(\alpha/\beta)$$

Similarly, $\quad B = 2\tan^{-1}(\alpha\beta)$

$\therefore$ Required area $= \frac{1}{2}bc\sin A$

$$= \frac{1}{2}bc\frac{2\tan(A/2)}{1 + \tan^2(A/2)} = \frac{bc(\alpha/\beta)}{1 + (\alpha/\beta)^2} = \frac{bc\alpha\beta}{\alpha^2 + \beta^2}$$

12. ***(a, b, c, d)*** Given, $|\mathbf{a}| = |\mathbf{b}| = |\mathbf{c}| = 1$

Angle between $\mathbf{a}$ and $\mathbf{c}$ = Angle between $\mathbf{b}$ and $\mathbf{c}$

$$\Rightarrow \quad \frac{\mathbf{a}\cdot\mathbf{c}}{|\mathbf{a}||\mathbf{c}|} = \frac{\mathbf{b}\cdot\mathbf{c}}{|\mathbf{b}||\mathbf{c}|} = \cos\theta$$

$$\Rightarrow \quad \mathbf{a}\cdot\mathbf{c} = \mathbf{b}\cdot\mathbf{c} = \cos\theta$$

Since, $\quad \mathbf{a}\cdot\mathbf{b} = 0$

and $\quad \mathbf{c} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma(\mathbf{a} \times \mathbf{b})$

$\therefore \quad \mathbf{a}\cdot\mathbf{c} = \alpha(\mathbf{a}\cdot\mathbf{a}) + \beta(\mathbf{a}\cdot\mathbf{b}) + \gamma\,\mathbf{a}\cdot(\mathbf{a}\times\mathbf{b})$

$\Rightarrow \quad \mathbf{a}\cdot\mathbf{c} = \alpha\cdot 1 + 0 + 0$

$\Rightarrow \quad \alpha = \cos\theta$

Similarly, $\mathbf{b}\cdot\mathbf{c} = \cos\theta = \beta$

$\therefore \quad \alpha = \beta = \cos\theta$

Again $\quad \mathbf{c}\cdot\mathbf{c} = 2\alpha^2 + \gamma^2 |\mathbf{a}\times\mathbf{b}|^2$

$\Rightarrow \quad 1 = 2\alpha^2 + \gamma^2\{|\mathbf{a}|^2|\mathbf{b}|^2 - (\mathbf{a}\cdot\mathbf{b})^2\}$

$\Rightarrow \quad 1 = 2\alpha^2 + \gamma^2$

$\therefore \quad \gamma^2 = 1 - 2\alpha^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

$\Rightarrow \quad \alpha^2 = \beta^2 = \dfrac{1-\gamma^2}{2} = \dfrac{1+\cos 2\theta}{2}$

13. ***(a, b, c)*** Given that g is the inverse of f.

$\therefore fog(x) = x$ for all $x \Rightarrow f'(g(x))\,g'(x) = 1$ for all x

$\Rightarrow \quad f'(g(1))\,g'(1) = 1$

$\Rightarrow \quad g'(1) = \dfrac{1}{f'(g(1))} \qquad \ldots(i)$

Again, g is the inverse of f.

$\therefore \quad f(x) = y \Rightarrow g(y) = x$

We have, $\quad f(0) = 1$

$\therefore \quad g(1) = 0$

From Eq. (i), we get

$$g'(1) = \frac{1}{f'(0)}$$

Now, $\quad f(x) = x^3 + e^{x/2}$

$\Rightarrow \quad f'(x) = 3x^2 + \dfrac{1}{2}e^{x/2} \Rightarrow f'(0) = \dfrac{1}{2}$

$\Rightarrow \quad g'(1) = 2$

14. ***(0)*** Given, $\tan(2\pi|\sin\theta|) = \tan\left(\dfrac{\pi}{2} - |\cos\theta|2\pi\right)$

$\Rightarrow \quad 2\pi|\sin\theta| = n\pi + \dfrac{\pi}{2} - |\cos\theta|2\pi$

$\Rightarrow \quad 2\pi(|\sin\theta| + |\cos\theta|) = n\pi + \dfrac{\pi}{2}$

$\Rightarrow \quad |\sin\theta| + |\cos\theta| = \dfrac{n}{2} + \dfrac{1}{4} \qquad \ldots(i)$

Since, $\quad 1 \le |\sin\theta| + |\cos\theta| \le \sqrt{2}$

$\therefore \quad 1 \le \dfrac{n}{2} + \dfrac{1}{4} \le \sqrt{2}$

$\Rightarrow \quad 4 \le 2n + 1 \le 4\sqrt{2}$

$\Rightarrow \quad \dfrac{3}{2} \le n \le \dfrac{4\sqrt{2}-1}{2}$

Thus, $n = 2$ is only possible value.

On putting the value of n in Eq. (i), we get

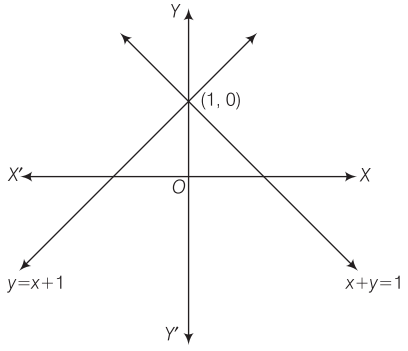
$$|\sin\theta| + |\cos\theta| = \frac{5}{4}$$

$\therefore \quad \lim\limits_{x\to\infty}\left[\dfrac{2}{f(x)}\right] = \lim\limits_{x\to\infty}\left[2\left(\dfrac{4}{5}\right)^x\right] = 0$

15. ***(2)*** Given, $x^2 - y^2 + 2y = 1$

$\Rightarrow \quad x^2 = y^2 - 2y + 1$

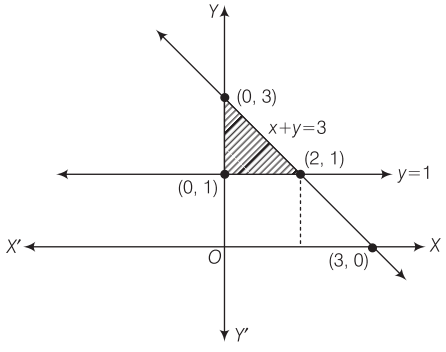
$\Rightarrow \quad x = \pm(y-1)$



Bisector of above lines are $x = 0$ and $y = 1$.

So, area between $x = 0$, $y = 1$ and $x + y = 3$ is the shaded region shown in figure.

The area is given by $\dfrac{1}{2}\times 2\times 2 = 2$ sq units.



16. ***(4)*** Out of 31 ways, only one is correct choice and other 30 choices are incorrect.

Let E be the event of answering correctly in r trails.

Then, E' is the event of answering incorrectly in r trails.

Now, $\quad P(E') = \dfrac{{}^{30}C_r}{{}^{31}C_r}$

$\therefore \quad P(E) = 1 - P(E') = 1 - \dfrac{{}^{30}C_r}{{}^{31}C_r}$

According to the question,

$$1 - \frac{{}^{30}C_r}{{}^{31}C_r} > \frac{1}{8}$$

$\Rightarrow \quad 1 - \dfrac{30!}{r!(30-r)!}\cdot\dfrac{r!(31-r!)}{31!} > \dfrac{1}{8}$

$\Rightarrow \quad 1 - \dfrac{31-r}{31} > \dfrac{1}{8} \Rightarrow \dfrac{r}{31} > \dfrac{1}{8} \Rightarrow r > \dfrac{31}{8}$

$\therefore$ Least value of r is 4.

17. ***(7)*** Given curve is $y = 2x^4 - x^2$.

$\Rightarrow \quad \dfrac{dy}{dx} = 8x^3 - 2x$

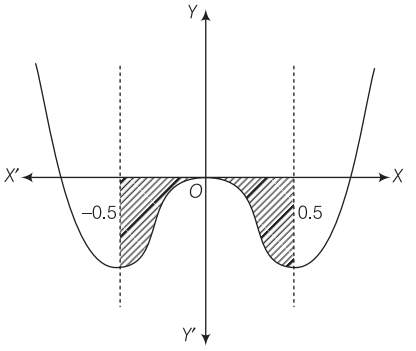
For maxima or minima, put $\dfrac{dy}{dx} = 0$

$\therefore \quad 8x^3 - 2x = 0 \Rightarrow x = -\dfrac{1}{2}, 0, \dfrac{1}{2}$

Now, $\quad \dfrac{d^2y}{dx^2} = 24x^2 - 2$

$\therefore \quad \left(\frac{d^2y}{dx^2}\right)_{x=0} = -2$ and $\left(\frac{d^2y}{dx^2}\right)_{x=\pm 1/2} = 4$

$\Rightarrow$ Curve is minimum at $x = -\frac{1}{2}$ and $\frac{1}{2}$.



$\therefore$ Required area

$$= \left|\int_{-1/2}^{1/2} (2x^4 - x^2)\,dx\right| = 2\left|\int_0^{1/2} (2x^4 - x^2)\,dx\right|$$

$$= \left|2\left[\frac{2x^5}{5} - \frac{x^3}{3}\right]_0^{1/2}\right| = \left|2\left[\frac{1}{80} - \frac{1}{24}\right]\right| = \left|\frac{1}{40} - \frac{1}{12}\right|$$

$$= \left|\frac{3-10}{120}\right|$$

$$= \frac{7}{120}\text{ sq unit}$$

On comparing, we get $k = 7$

18. *(3)* Let $f(x) = 7 + 2x\log_e 25 - 5^{x-1} - 5^{2-x}$

$$= 7 + 4x\log_e 5 - \frac{5^x}{5} - \frac{25}{5^x}$$

$$\Rightarrow \quad f'(x) = 4\log_e 5 - \frac{5^x}{5}\log_e 5 + 25\cdot 5^{-x}\log_e 5$$

$$= \frac{\log_e 5}{5}[20 - 5^x + 125.5^{-x}]$$

$$= \frac{-\log_e 5}{5}\left[\frac{5^{2x} - 20\cdot 5^x - 125}{5^x}\right]$$

$$= \frac{-\log_e 5}{5^{x+1}}[(5^x - 25)(5^x + 5)]$$

For maxima or minima, put $f'(x) = 0$

$\therefore \quad (5^x - 25)(5^x + 5) = 0$

$\Rightarrow \quad 5^x - 25 = 0$ or $5^x = -5$ is not possible

$\Rightarrow \quad x = 2$

Now, $f''(x) = \frac{\log_e 5}{5}[0 - 5^x\log_e 5 - 125\cdot 5^{-x}\log_e 5] < 0$

[as 5^x and 5^{-x} are positive]

Thus, at $x = 2$, $f(x)$ is maximum, so $a = 2$.

Also, $r = \lim_{x\to 0}\int_0^x \frac{t^2\,dt}{\{x^2\tan(\pi + x)\}}$

$$= \lim_{x\to 0}\frac{x^3}{3x^2\tan(\pi + x)} = \lim_{x\to 0}\frac{x}{3\tan x} = \frac{1}{3}$$

$\Rightarrow \quad r = \frac{1}{3}$

Therefore, the required GP is $2 + \frac{2}{3} + \frac{2}{3} + \frac{2}{9} + \ldots$

whose sum is $\frac{2}{1 - \frac{1}{3}} = 3$.

19. *(4)* We have, $f(x) = \sin^{-1}|\sin px| + \cos^{-1}(\cos px)$

Let $g(x) = \sin^{-1}|\sin x| + \cos^{-1}(\cos x)$

$$\Rightarrow \quad g(x) = \begin{cases} 2x & , 0 \le x \le \frac{\pi}{2} \\ \pi & , \frac{\pi}{2} < x \le \frac{3\pi}{2} \\ 4\pi - 2x, & \frac{3\pi}{2} < x \le 2\pi \end{cases}$$

$g(x)$ is periodic with period 2π and is constant in the continuous interval $\left[2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right]$

where, $(n \in I)$ and $f(x) = g(px)$.

So, $f(x)$ is constant in the interval $\left[\frac{2n\pi}{p} + \frac{\pi}{2p}, \frac{2n\pi}{p} + \frac{3\pi}{2p}\right]$

$$\Rightarrow \quad \frac{\pi}{4} = \frac{3\pi}{2p} - \frac{\pi}{2p} \Rightarrow \frac{\pi}{p} = \frac{\pi}{4}$$

$$\Rightarrow \quad p = 4$$

20. *(5)* The five possible ways of inviting 4 ladies and 4 gentlemen for the party with the help of the following table.

Man's relative		Wife's relative		Number of ways
Ladies (5)	Gentlemen (3)	Ladies (3)	Gentlemen (5)	
4	0	0	4	${}^5C_4 \times {}^3C_0 \times {}^3C_0 \times {}^5C_4 = 25$
3	1	1	3	${}^5C_3 \times {}^3C_1 \times {}^3C_1 \times {}^5C_3 = 900$
2	2	2	2	${}^5C_2 \times {}^3C_2 \times {}^3C_2 \times {}^5C_2 = 900$
1	3	3	1	${}^5C_1 \times {}^3C_3 \times {}^3C_3 \times {}^5C_1 = 25$

$\therefore$ Required number of ways to invite

$$\lambda = 25 + 900 + 900 + 25 = 1850$$

Hence, $\lambda - 1845 = 1850 - 1845 = 5$

21. *(2)* Since, $3^{2\sin 2x - 1}$, 14 and $3^{4 - 2\sin 2x}$ are in AP.

$$\therefore \quad 28 = 3^{2\sin 2x}\cdot\frac{1}{3} + 3^4\cdot\frac{1}{3^{2\sin 2x}}$$

Put $\quad 3^{2\sin 2x} = t \Rightarrow 28 = \frac{t}{3} + \frac{81}{t}$

$$\Rightarrow \quad 84t = t^2 + 243$$

$\Rightarrow t^2 - 84t + 243 = 0 \Rightarrow (t - 81)(t - 3) = 0 \Rightarrow t = 3, 81$

$\therefore 3^{2\sin 2x} = 3^1, 3^4 \Rightarrow 2\sin 2x = 1, 4 \Rightarrow \sin 2x = \frac{1}{2}, 2$

$\Rightarrow \sin 2x = \frac{1}{2}$ [$\because \sin 2x \ne 2$]

$\therefore$ First term $= 3^{1-1} = 1$

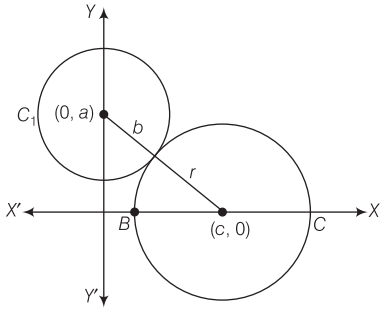
Second term $= 14$ and third term $= 27$

Here, common difference $= 13$

$\therefore$ 10th term $= a + 9d = 1 + 9\times 13 = 118 = 59\,k$

Hence, $\quad k = 2$

22. ***(a)*** From figure, $\sqrt{c^2+a^2}=b+r$



Consider the point $P(0,p)$

On the X-axis, $B(c-r,0)$ and $C(c+r,0)$

Now, slope of $BP=\dfrac{-p}{c-r}$, slope of $CP=\dfrac{-p}{c+r}$

If $\angle BPC=\theta$, then

$$\tan\theta=\left|\frac{\dfrac{p}{c-r}-\dfrac{p}{c+r}}{1+\dfrac{p^2}{c^2-r^2}}\right|=\left|\frac{2pr}{c^2-r^2+p^2}\right|$$

23. ***(b)*** According to the given condition, θ is a constant for any choice c.

$$\therefore \quad \frac{2pr}{c^2-r^2+p^2}=\text{Constant}$$

$$\Rightarrow \quad \frac{r}{c^2-r^2+p^2}=\text{Constant}$$

$$\Rightarrow \quad \frac{\sqrt{c^2+a^2}-b}{c^2-(\sqrt{c^2+a^2}-b)^2+p^2}=\text{Constant}\ [\because \sqrt{c^2+a^2}=b+r]$$

$$\Rightarrow \quad \frac{\sqrt{c^2+a^2}-b}{2b\sqrt{c^2+a^2}-a^2-b^2+p^2}=\text{Constant}$$

$$\Rightarrow \quad \frac{\sqrt{c^2+a^2}-b}{\sqrt{c^2+a^2}-\lambda}=\text{Constant}$$

where, $\dfrac{a^2+b^2-p^2}{2b}=\lambda$

which is possible only if $\lambda=b$, i.e. $\dfrac{a^2+b^2-p^2}{2b}=b$

or $\quad p=\pm\sqrt{a^2-b^2}$

$\therefore \quad P=(0,\ \pm\sqrt{a^2-b^2})$

Sol. (Q. Nos. 24-25) Vectors equation of lines L_1 and L_2 are

$$\mathbf{r}=-\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}+\lambda(3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}})$$

and $\quad \mathbf{r}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+3\hat{\mathbf{k}}+\mu(\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}})$

Let $\quad \mathbf{n}_1=3\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$ and $\mathbf{n}_2=\hat{\mathbf{i}}+2\hat{\mathbf{j}}+3\hat{\mathbf{k}}$

Now, $\quad \mathbf{n}_1\times\mathbf{n}_2=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 3 & 1 & 2\\ 1 & 2 & 3\end{vmatrix}$

$$=-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}$$

24. ***(c)*** Unit vector perpendicular to L_1 and L_2

$$=\frac{\mathbf{n}_1\times\mathbf{n}_2}{|\mathbf{n}_1\times\mathbf{n}_2|}=\frac{-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{\sqrt{75}}=\frac{-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}}{5\sqrt{3}}$$

25. ***(a)*** Since, $\mathbf{n}_1\times\mathbf{n}_2$ is perpendicular to both L_1 and L_2

and $\quad \mathbf{n}_1\times\mathbf{n}_2=-\hat{\mathbf{i}}-7\hat{\mathbf{j}}+5\hat{\mathbf{k}}$

Let $\quad \mathbf{a}=-\hat{\mathbf{i}}-2\hat{\mathbf{j}}-\hat{\mathbf{k}}$

$P\equiv(1,1,1)$, then $\mathbf{OP}=\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}=\vec{\alpha}$

$\therefore \quad \mathbf{a}\cdot\mathbf{n}=1+14-5=10$

and $\quad \vec{\alpha}\cdot\mathbf{n}=-1-7+5=-3$

Equation of the plane through point $\mathbf{a}$ and normal to L_1 and L_2, i.e. normal to $\mathbf{n}$ is $\mathbf{r}\cdot\mathbf{n}=\mathbf{a}\cdot\mathbf{n}$.

$\Rightarrow \quad \mathbf{r}\cdot\mathbf{n}=10$

$\Rightarrow \quad \mathbf{r}\cdot\mathbf{n}-10=0$ ...(i)

$\therefore$ Length of perpendicular from $P(1,1,1)$ to plane (i) is

$$d=\frac{|\vec{\alpha}\cdot\mathbf{n}-10|}{|\mathbf{n}|}=\left|\frac{-3-10}{\sqrt{75}}\right|=\frac{13}{\sqrt{75}}\ \text{units}$$

26. ***(a)*** We have, $\dfrac{dP(x)}{dx}>P(x)$

$$\Rightarrow \quad e^{-x}\frac{dP(x)}{dx}-e^{-x}P(x)>0$$

$$\Rightarrow \quad \frac{d}{dx}(P(x)e^{-x})>0$$

$\therefore P(x)e^{-x}$ is an increasing function.

$$\Rightarrow \quad P(x)\,e^{-x}>P(1)\,e^{-1},\ \forall\ x\geq 1$$

$$\Rightarrow \quad P(x)\ e^{-x}>0,\ \forall\ x>1$$

$$\Rightarrow \quad P(x)>0,\ \forall\ x>1$$

27. ***(b)*** Given, $\dfrac{d}{dx}\phi(x)>2ax\ \phi(x)$

$$\Rightarrow \quad e^{-ax^2}\frac{d}{dx}\phi(x)-e^{-ax^2}2ax\phi(x)>0$$

$$\Rightarrow \quad \frac{d}{dx}\{\phi(x)\ =e^{-ax^2}\}>0$$

$\therefore \quad \phi(x)\ e^{-ax^2}$ is an increasing function.

But $\phi(x_0)=0$ and e^{-ax^2} is always positive.

$\Rightarrow \quad \phi(x)>0$ for all $x>x_0$

$\therefore \quad \phi(x)$ cannot be zero for any $x>x_0$.

28. A $\to$ r, B $\to$ s, C $\to$ q, D $\to$ p

A. $\lim\limits_{n\to\infty} n\int_0^{\pi/2}(1-\sqrt[n]{\sin x})\,dx$

$$=\int_0^{\pi/2}\lim_{n\to\infty}\left(\frac{1-\sqrt[n]{\sin x}}{1/n}\right)dx$$

$$=\int_0^{\pi/2}\lim_{n\to\infty}\frac{0-(\sin x)^{1/n}\left(-\dfrac{1}{n^2}\right)\log\sin x}{-\dfrac{1}{n^2}}\,dx$$

$$=\int_0^{\pi/2}\log\sin dx=-\frac{\pi}{2}\log 2 \qquad \left[\because \lim_{n\to\infty}(\sin x)^{1/n}=1\right]$$

$$\therefore \quad -\frac{\pi}{2}\log 2=\frac{k\pi}{2}\log 2$$

$$\Rightarrow \quad k=-1$$

B. We have, $f'(x)=\dfrac{g'(x)}{\sqrt{a+g^3(x)}}$

and $g'(x)=(1+\sin(\cos^2 x))(-\sin x)$

Hence, $f'(x)=\dfrac{(1+\sin x(\cos^2 x))(-\sin x)}{\sqrt{1+g^3(x)}}$

$\Rightarrow f'\left(\dfrac{\pi}{2}\right)=\dfrac{1+0}{\sqrt{1+g^3\left(\dfrac{\pi}{2}\right)}}=-1,\ g\left(\dfrac{\pi}{2}\right)=0$

$\Rightarrow \quad 2f'\left(\dfrac{\pi}{2}\right)=-2$

C. $\lim\limits_{n\to\infty}\left\{\left(\dfrac{1}{n}\right)^1\cdot\left(\dfrac{2}{n}\right)^2\cdot\left(\dfrac{3}{n}\right)^3\cdots\left(\dfrac{n}{n}\right)\right\}^{\frac{1}{n^2}}$

$\Rightarrow \log L=\lim\limits_{h\to\infty}\dfrac{1}{n^2}\left\{1\log\left(\dfrac{1}{n}\right)+2\log\left(\dfrac{2}{n}\right)+3\log\left(\dfrac{3}{n}\right)+\ldots+n\log\left(\dfrac{n}{h}\right)\right\}$

$\because$ General term of $\log L=\dfrac{r}{n^2}\log\left(\dfrac{r}{n}\right)$

$\therefore$ Sum $=\dfrac{1}{n}\sum\limits_{r=1}^{n}\dfrac{r}{n}\log\left(\dfrac{r}{n}\right)$

$\log L=\int_0^1 x\log x\,dx=\left[\dfrac{x^2\cdot\log x}{2}\right]_0^1-\dfrac{1}{2}\int_0^1 x^2\cdot\dfrac{1}{x}dx$

$=\left[0-\dfrac{1}{2}\cdot\dfrac{x^2}{2}\right]_0^1=-\dfrac{1}{4}$

$\therefore \quad L=e^{-1/4}$

D. Let $\quad I=\int_0^{\pi}\dfrac{x|\sin x|}{1+|\cos x|}dx \qquad \ldots(i)$

$\Rightarrow \quad I=\int_0^{\pi}\dfrac{(\pi-x)|\sin(\pi-x)|}{1+|\cos(\pi-x)|}dx$

$\Rightarrow \quad I=\int_0^{\pi}\dfrac{(\pi-x)|\sin x|}{1+|\cos x|}dx \qquad \ldots(ii)$

On adding Eqs. (i) and (ii), we get

$2I=\pi\int_0^{\pi}\dfrac{|\sin x|}{1+|\cos x|}dx$

$=\pi\cdot 2\int_0^{\pi}\dfrac{|\sin x|}{1+|\cos x|}dx=2\pi\int_0^{\pi/2}\dfrac{\sin x}{1+\cos x}dx$

$=-2\pi[\log(1+\cos x)]_0^{\pi/2}=-2\pi[\log 1-\log 2]$

$\Rightarrow \quad 2I=2\pi\log 2$

$\therefore \quad I=\pi\log 2=b\pi\log 2\Rightarrow b=1$

29. A $\to$ r, B $\to$ p, C $\to$ s, D $\to$ q

A. For $n=2,(A^{-1}BA)(A^{-1}BA)=A^{-1}B^2A$

For $n=3,(A^{-1}BA)=(A^{-1}BA)^2(A^{-1}BA)$

$=(A^{-1}B^2A)(A^{-1}BA)=A^{-1}B^3A$

and so on.

Thus, $(A^{-1}BA)^n=A^{-1}B^nA$

B. $A*(B+C)=\dfrac{1}{2}\{A(B+C)'+A'(B+C)\}$

$=\dfrac{1}{2}\{A(B'+C')+A'(B+C)\}$

$=\dfrac{1}{2}(AB'+A'B)+\dfrac{1}{2}(AC'+A'C)=A*B+A*C$

C. Since, $A^2=2A-I$

$\Rightarrow A^3=A^2\cdot A=2A^2-IA$

$=2A^2-A=2(2A-I)-A=3A-2I$

$=3A-(3-1)I$

$\vdots \quad \vdots \quad \vdots$

$A^n=nA-(n-1)I$

D. $A^2=A\cdot A=\begin{bmatrix}1&0\\1&1\end{bmatrix}\begin{bmatrix}1&0\\1&1\end{bmatrix}=\begin{bmatrix}1&0\\2&1\end{bmatrix}$

$A^3=A^2\cdot A=\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}1&0\\1&1\end{bmatrix}=\begin{bmatrix}1&0\\3&1\end{bmatrix}$

$\vdots \quad \vdots \quad \vdots$

$\therefore \quad A^n=\begin{bmatrix}1&0\\n&1\end{bmatrix}$

$\therefore A^{-n}=\begin{bmatrix}1&0\\-n&1\end{bmatrix}\Rightarrow\lim\limits_{n\to\infty}\dfrac{A^{-n}}{n}=\begin{bmatrix}\lim\limits_{n\to\infty}\dfrac{1}{n}&0\\\lim\limits_{n\to\infty}\dfrac{-n}{n}&\lim\limits_{n\to\infty}\dfrac{1}{n}\end{bmatrix}$

and $\lim\limits_{n\to\infty}\dfrac{A^{-n}}{n^2}=\begin{bmatrix}\lim\limits_{n\to\infty}\dfrac{1}{n^2}&0\\\lim\limits_{n\to\infty}\left(-\dfrac{1}{n}\right)&\lim\limits_{n\to\infty}\dfrac{1}{n^2}\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$

30. A $\to$ r, B $\to$ s, C $\to$ q, D $\to$ p

A. On eliminating θ from $x=1+4\cos\theta$ and $y=2+3\sin\theta$

we get $\dfrac{(x-1)^2}{16}+\dfrac{(y-2)^2}{9}=1$

Hence, $a=4$ and $e=\dfrac{\sqrt{7}}{4}$

$\Rightarrow ae=\sqrt{7}$

$\therefore$ Distance between the foci $=2\sqrt{7}$

B. We have, $\dfrac{1}{e_1^2}+\dfrac{1}{e_2^2}=1$

$\Rightarrow\dfrac{(e_1+e_2)^2-2e_1e_2}{e_1^2e_2^2}=1$

$\Rightarrow\dfrac{a^2-4}{4}=1$

$\Rightarrow a=\pm 2\sqrt{2}$

C. Equation of the normal at (c,c) to the rectangular hyperbola $xy=c^2$ is $y-ct=t^2(x-ct)$

which passes through $(ct',\ c/t')$, if

$ct'=c/t'\Rightarrow t'=-1$

D. Let the normals at P and Q meet on the parabola $y^2=4ax$ at $(aT^2,2aT)$.

Then, $T=-t_1-\dfrac{2}{t_1}$ and $T=-t_2-\dfrac{2}{t_2}$

$\therefore -t_1-\dfrac{2}{t_1}=-t_2-\dfrac{2}{t_2}$

$\Rightarrow(t_2-t_1)=2\left(\dfrac{1}{t_1}-\dfrac{1}{t_2}\right)$

$\Rightarrow \quad t_1t_2=2 \qquad [\because t_1\neq t_2]$

TARGET WBJEE 2017

TEST RIDER

Comprehensive Simulator Test Series for WBJEE

WBJEE

(PAPER ONE PREP UP)

FULL TEST

(COMPLETE SYLLABUS)
FULLY SOLVED

Mock Questions from Complete Syllabus with Complete Solutions

INSTRUCTIONS

- This test consists of 75 questions and each question is allotted 1 mark for correct response.
- Candidates will be awarded marks as stated above for correct response of each question. 1/4 mark will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. A point on the ellipse $4x^2 + 9y^2 = 36$, where the tangent is equally inclined to the axes

(a) $\left(\frac{9}{\sqrt{13}}, \frac{9}{\sqrt{13}}\right)$ (b) $\left(\frac{9}{\sqrt{13}}, \frac{-9}{\sqrt{13}}\right)$

(c) $\left(\frac{9}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right)$ (d) $\left(\frac{7}{\sqrt{13}}, \frac{-5}{\sqrt{13}}\right)$

2. If $z^2 + z + 1 = 0$, where z is complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2$ is

(a) 18 (b) 12 (c) 54 (d) 6

3. If a rod of length l slides with its ends on two perpendicular lines, then the locus of its mid-point is

(a) $4x^2 + 4y^2 = l^2$ (b) $x^2 + 2y^2 = l^2$

(c) $x^2 + y^2 = l^2$ (d) $2x^2 + 2y^2 = l^2$

4. The value of $\lim_{x \to 0} \frac{(1 - \cos 2x)\sin 7x}{x^2 \sin 5x}$ is

(a) $\frac{5}{14}$ (b) $\frac{14}{5}$ (c) $\frac{10}{14}$ (d) $\frac{14}{10}$

5. The equation $\frac{x^2}{1-k} - \frac{y^2}{1+k} = 1, |k| < 1$, represents a/an

(a) circle (b) ellipse

(c) hyperbola (d) None of these

6. An aeroplane flying at a height of 300 m above the ground passes vertically above another plane at an instant when the angles of elevation of two planes from the same point on the ground are 60° and 45°, respectively. What is the height of the lower plane from the ground?

(a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$ m

(c) $150(\sqrt{3} + 1)$ m (d) 500 m

7. The value of $|\mathbf{p} \times \hat{\mathbf{i}}|^2 + |\mathbf{p} \times \hat{\mathbf{j}}|^2 + |\mathbf{p} \times \hat{\mathbf{k}}|^2$ is

(a) p^2 (b) $2p^2$ (c) $3p^2$ (d) $4p^2$

8. For a real number y, let $[y]$ denotes the greatest integer less than or equal to y. Then, the function $f(x) = \frac{\tan \pi [(x - \pi)]}{1 + [x]^2}$

(a) discontinuous at some x

(b) continuous at all x, but the derivative $f'(x)$ does not exist for some x

(c) $f'(x)$ exists for all x

(d) $f'(x)$ exists for all x, but the derivative $f''(x)$ does not exist for some x

9. Which of the following functions is an even function?

(a) $f(x) = \frac{a^x + 1}{a^x - 1}$ (b) $f(x) = x\frac{a^x - 1}{a^x + 1}$

(c) $f(x) = \log_2(x + \sqrt{x^2 + 1})$ (d) $f(x) = \frac{a^x + a^{-x}}{a^x - a^{-x}}$

10. The number of solutions of the equation $\sin 5x \cos 3x = \sin 6x \cos 2x$ in $[0, \pi]$ is

(a) 2 (b) 3 (c) 4 (d) 5

11. Let the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$. Then, (α, β) is equal to

(a) $(5, -15)$ (b) $(-5, 5)$ (c) $(-6, 7)$ (d) $(6, -17)$

12. If $5^x + (2\sqrt{3})^{2x} \geq 13^x$, then the solution set for x is
(a) $\{2\}$ (b) $[0,2]$ (c) $[2, \infty)$ (d) $(-\infty, 2]$

13. $\int \frac{\log(x/e)}{(\log x)^2} dx$ is equal to
(a) $\frac{x}{\log x} + C$ (b) $\frac{\log x}{x} + C$
(c) $\frac{x}{(\log x)^2} + C$ (d) None of these

14. If $A = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix P and $|P| = 4$, then α is equal to
(a) 0 (b) 4 (c) 5 (d) 11

15. There are three vectors $\mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$, $\mathbf{q} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and $\mathbf{r} = x\mathbf{p} + y\mathbf{q}$. If the vectors $\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$, $3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\mathbf{r}$ are coplanar, then $\frac{x}{y}$ is equal to
(a) -1 (b) -2 (c) -3 (d) $\frac{2}{3}$

16. The statement $p \to (q \to p)$ is equivalent to
(a) $p \to (p \to q)$ (b) $p \to (p \wedge q)$
(c) $p \to (p \leftrightarrow q)$ (d) $p \to (p \vee q)$

17. The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green balls, is
(a) 84 (b) 86 (c) 270 (d) 286

18. If two different tangents of $y^2 = 4x$ are the normals to the parabola $x^2 = 4ay$, then
(a) $|a| < \frac{1}{2\sqrt{2}}$ (b) $|a| < \frac{1}{\sqrt{2}}$
(c) $|a| > \frac{1}{2\sqrt{2}}$ (d) $|a| > \frac{1}{\sqrt{2}}$

19. The difference between the greatest and least values of the function $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$, is
(a) $\frac{2}{3}$ (b) $\frac{8}{7}$ (c) $\frac{9}{4}$ (d) $\frac{3}{8}$

20. If $\begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_k \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{k+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{k+4} \end{vmatrix} = 0$, then k equals
(a) 4 (b) 6
(c) 5 (d) None of these

21. If the product of n positive numbers is 1, then their sum is positive integer, which is
(a) equal to $n + n^2$ (b) divisible by n
(c) never less than n (d) equal to 1

22. In the binomial expansion of $(a-b)^n$, $n \geq 5$, if the sum of 5th and 6th terms is zero, then $\frac{a}{b}$ equals
(a) $\frac{n-4}{5}$ (b) $\frac{5}{n-4}$ (c) $\frac{6}{n-5}$ (d) $\frac{n-5}{6}$

23. The value of $\lim_{n\to\infty}\left(\frac{a-1+\sqrt[n]{b}}{a}\right)^n$; $a, b > 0$ is
(a) $\sqrt[b]{a}$ (b) $\sqrt[a]{b}$ (c) $\sqrt{a}$ (d) $\sqrt{b}$

24. The area bounded by the curves $y = \begin{cases} x^{1/\ln x}, & x \neq 1 \\ e, & x \neq 1 \end{cases}$ and $y = |x - e|$ is
(a) e^2 sq units (b) $2e^2$ sq units
(c) 1 sq unit (d) $\frac{e^2}{2}$ sq units

25. The probability that certain electronic component fails when used for the first time is 0.10. If it does not fail immediately, then the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is
(a) 0.1 (b) 0.9 (c) 0.891 (d) 0.119

26. A point P moves in such a way that the ratio of its distance from two coplanar points is always a fixed number $(\neq 1)$. Then, its locus is a
(a) circle (b) parabola
(c) pair of straight lines (d) hyperbola

27. If the position vectors of A, B and C are respectively, $2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$ and $3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$, then $\cos^2 A$ is equal to
(a) 1 (b) 0 (c) $\frac{6}{41}$ (d) $\frac{35}{41}$

28. The radius of the circle, which is touched by the line $y = x$ and has its centre on the positive direction of X-axis and also cut-off a chord of length 2 units along the line $\sqrt{3}y - x = 0$, is
(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{5}$

29. The locus represented by $|z-1| = |z+i|$ is a/an
(a) line through the origin
(b) circle of radius 1
(c) circle on the line join of 1 and $-i$ as a diameter
(d) ellipse with foci at 1 and $-i$

30. The area of the triangle formed by the tangent to the curve $y = \frac{8}{(4+x^2)}$ at $x = 2$ and the coordinate axes is
(a) 4 sq units (b) $\frac{7}{2}$ sq units (c) 2 sq units (d) 8 sq units

31. If $\mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and the vector $\mathbf{c}$ such that $\mathbf{a} \cdot \mathbf{c} = 4$ and $\mathbf{a} \times \mathbf{c} = \mathbf{b}$, is
(a) $3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ (b) $2\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (c) $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ (d) $\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + \hat{\mathbf{k}}$

32. The sum of n terms of the series $\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots$ upto n terms equals
(a) $\frac{n}{2}\left(\log \frac{a}{b} + n \log ab\right)$ (b) $n\left(\frac{n}{2}\log \frac{a}{b} + \log ab\right)$
(c) $\frac{n}{2}\left(n \log \frac{a}{b} + \log ab\right)$ (d) $n\left(\log \frac{a}{b} + \frac{n}{2}\log ab\right)$

33. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = k^2$ orthogonally, then the equation of the locus of its centre is
(a) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - k^2) = 0$
(b) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - k^2) = 0$
(c) $2ax + 2by - (a^2 + b^2 + k^2) = 0$
(d) $2ax + 2by - (a^2 - b^2 + k^2) = 0$

34. If $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ are the unit vectors and θ is the acute angle between them, then $2\hat{\mathbf{a}} \times 3\hat{\mathbf{b}}$ is a unit vector for
(a) exactly one value of θ *(b)* exactly two values of θ
(c) more than two values of θ *(d)* no vlaue of θ

35. If $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$, $\mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{c} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$, then the area of parallelogram having diagonals $\mathbf{a} + \mathbf{b}$ and $\mathbf{b} + \mathbf{c}$ is
(a) $\sqrt{6}$ sq units *(b)* $\frac{\sqrt{6}}{2}$ sq units
(c) $\frac{1}{2}\sqrt{21}$ sq units *(d)* $4\sqrt{6}$ sq units

36. For positive numbers x, y and z, the numerical value of $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is
(a) 0 *(b)* 1 *(c)* 2 *(d)* 4

37. Let $y = f(a^x)$ and $f'(\sin x) = \log_e x$. Then, for $\frac{\pi}{2} < x < \pi$, $\frac{dy}{dx}$ is equal to
(a) $a^x \log_e a \log_e(\sin^{-1} a^x)$ *(b)* $a^x \log_e a \log_e(\pi - \sin^{-1} a^x)$
(c) $a^x \log_e a \log_e(\pi + \sin^{-1} a^x)$ *(d)* None of these

38. If $x^2 + 5 = 2x - 4\cos(a + b)$, where $a, b \in (0, 5)$, is satisfied for atleast one real x, then the maximum value of $a + b$ in $[0, 2\pi]$ is
(a) 2π *(b)* π
(c) 3π *(d)* None of these

39. Under what condition straight line $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$ is parallel to the XY-plane?
(a) $l = 0, m = 0$ *(b)* $l = 0$ *(c)* $m = 0$ *(d)* $n = 0$

40. At a telephone enquiry system, the number of phone calls regarding relevant equiry follows the Poisson distribution with average 5 phone calls during 10 min time intervals. The probability that there is Most one phone call during a 10 min time period, is
(a) $\frac{5}{6}$ *(b)* $\frac{6}{55}$ *(c)* $\frac{6}{e^2}$ *(d)* $\frac{6}{5^e}$

41. If $f(x) = \cos ax + \sin x$ is periodic, then a must be
(a) rational *(b)* irrational
(c) positive real number *(d)* None of these

42. The larger of $99^{50} + 100^{50}$ and 101^{50} is
(a) 101^{50} *(b)* $99^{50} + 100^{50}$
(c) Both are equal *(d)* None of these

43. If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the hyperbola $4x^2 - 9y^2 - 36 = 0$ are at right angles, then $\frac{x_1 x_2}{y_1 y_2}$ is equal to
(a) $\frac{81}{16}$ *(b)* $\frac{-81}{16}$ *(c)* $\frac{9}{4}$ *(d)* $\frac{-9}{4}$

44. How many different nine-digit numbers can be formed from the number 445577666 by rearranging its digits so that the odd digit occupy even positions?
(a) 180 *(b)* 60 *(c)* 36 *(d)* 16

45. The 99th term of the series $2 + 7 + 14 + 23 + 34 + \ldots$ is
(a) 10000 *(b)* 9998
(c) 9999 *(d)* None of these

46. The equation of chord of the circle $x^2 + y^2 = r^2$ passing through the point (2, 3) farthest from the centre is
(a) $3x - y = 3$ *(b)* $2x + 3y = 13$
(c) $x - y + 1 = 0$ *(d)* $x - 2y + 4 = 0$

47. The coordinates (x, y) of a point M satisfy the relations $3x + 4y = 9$ and $y = mx + 1$. The number of integral values of m for which x-coordinate of M is also an integer, is
(a) 1 *(b)* 0 *(c)* 4 *(d)* 2

48. $\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$ is equal to
(a) $\frac{3}{4}$ *(b)* $\frac{3}{2}$ *(c)* $\frac{3}{5}$ *(d)* $\frac{2}{5}$

49. The image of the parabola $y^2 = 4x$ about the line $x - y + 1 = 0$ is
(a) $(x - 1)^2 = 4(y + 1)$ *(b)* $(x + 1)^2 = 4(y - 1)$
(c) $(x - 1)^2 = -4(y - 1)$ *(d)* $(x + 1)^2 = -4(y + 1)$

50. The area bounded by $y = x|\sin x|$ and X-axis between $x = 0$, $x = 2\pi$ is
(a) π sq units *(b)* 2π sq units
(c) 3π sq units *(d)* 4π sq units

51. If $\lim_{n \to \infty} \frac{1 - (10)^n}{1 + (10)^{n+1}} = \frac{-\alpha}{10}$, then the value of α is
(a) -1 *(b)* 0 *(c)* 2 *(d)* 1

52. The negation of the proposition 'If 2 is prime, then 3 is odd' is
(a) 2 is prime and 3 is not odd
(b) if 2 is not prime, then 3 is odd
(c) if 2 is not prime, then 3 is not odd
(d) 2 is not prime and 3 is odd

53. The negation of the statement 'If I become a teacher, then I will open a school' is
(a) I will become a teacher and I will not open a school
(b) Either I will not become a teacher or I will not open a school
(c) Neither I will become a teacher nor I will open a school
(d) I will not become a teacher or I will open a school

54. If 6th term in the expansion of the binomial $[\sqrt{2^{\log(10-3^x)}} + \sqrt[5]{2^{(x-2)\log 3}}]^m$ is equal to 21 and it is known that the binomial coefficient of 2nd, 3rd and 4th term in the expansion represent respectively the 1st, 3rd and 5th terms of an AP, then x is equal to
(a) 3 *(b)* -2 *(c)* 0 *(d)* 1

55. The function $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos|x|$ is not differentiable at
(a) 0 *(b)* -1 *(c)* 2 *(d)* 1

56. The lines joining the origin to the points of intersection of the lines $4x + 3y = 24$ with the circle $(x-3)^2 + (y-4)^2 = 25$, are
(a) perpendicular *(b)* coincident
(c) equally inclined to X-axis *(d)* None of these

57. If the tangent to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at any point on it cuts the X and Y-axes respectively at A and B, then $OA + OB$ is equal to
(a) a *(b)* $\frac{a}{2}$ *(c)* $2a$ *(d)* None of these

58. Let $a = \min\{x^2 + 2x + 3, x \in R\}$ and $b = \lim_{\theta \to 0} \frac{1 - \cos\theta}{\theta^2}$. Then, the value of $\sum_{r=0}^{n} a^r \cdot b^{n-r}$ is
(a) $\frac{2^{n+1}+1}{3 \cdot 2^n}$ *(b)* $\frac{4^{n+1}-1}{3 \cdot 2^n}$ *(c)* $\frac{2^{n+1}-1}{3 \cdot 2^n}$ *(d)* None of these

59. The equation of the plane through the point $(2, -1, -3)$ and parallel to the lines $\frac{x-1}{3} = \frac{y+2}{2} = \frac{z}{-4}$ and $\frac{x}{2} = \frac{y-1}{-3} = \frac{z-2}{2}$ is
(a) $8x - 14y + 13z + 37 = 0$ *(b)* $8x + 14y + 13z + 37 = 0$
(c) $8x + 14y + 13z - 37 = 0$ *(d)* $8x + 14y - 13z + 37 = 0$

60. The total number of injections (one-one and into mappings) from $\{m_1, m_2, m_3, m_4\}$ to $\{n_1, n_2, n_3, n_4, n_5, n_6, n_7\}$ is
(a) 840 *(b)* 800 *(c)* 420 *(d)* 400

61. The value of $\left[(0.16)^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right)}\right]^{1/2}$ is
(a) 0 *(b)* -0.16
(c) 0.16 *(d)* None of these

62. If $y_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, then the value of $y_1 \cdot y_2 \cdot y_3 \ldots \infty$ is
(a) -1 *(b)* 1 *(c)* π *(d)* 0

63. The equation $k\cos x - 3\sin x = k + 1$ is solvable only, if
(a) $k \in (-\infty, 4]$ *(b)* $k \in [4, \infty)$
(c) $k \in [-4, 4]$ *(d)* None of these

64. If the circle $x^2 + y^2 + 4x + 22y + c = 0$ bisects the circumference of the circle $x^2 + y^2 - 2x + 8y - d = 0$, the $(c + d)$ is equal to
(a) 40 *(b)* 52 *(c)* 50 *(d)* 56

65. If one root of the equation $x^2 + px + q = 0$ is $2 + \sqrt{3}$, then the values of p and q are respectively
(a) $-2, -\sqrt{3}$ *(b)* $2, \sqrt{3}$ *(c)* $-4, 1$ *(d)* $4, -1$

66. A man has 7 relatives, 4 of them are ladies and 4 gentlemen while his wife has also 7 relatives, 3 of them are ladies and 4 gentlemen. In how many ways, can they invite a dinner party of 3 ladies and 3 gentlemen so that there are 3 of man's relative and 3 of wife's relative?
(a) 500 *(b)* 486 *(c)* 102 *(d)* 485

67. In a geometric progression consisting of positive terms, if each term equals the sum of next two terms, then the common ratio of this progression equals
(a) $\frac{1}{2}\sqrt{5}$ *(b)* $\frac{1}{2}(\sqrt{5}-1)$ *(c)* $\frac{1}{2}(1-\sqrt{5})$ *(d)* $\sqrt{5}$

68. The interval in which $2x^3 + 5$ increases less rapidly than $9x^2 - 12x$, is
(a) $(2, \infty)$ *(b)* $(1, 2)$
(c) $(-\infty, 1)$ *(d)* None of these

69. The area of the region bounded by the curve $y = xe^{|x|}$ and lines $|x| = 1$, $y = 0$ is
(a) 2 sq units *(b)* 4 sq units *(c)* 1 sq unit *(d)* 6 sq units

70. $\int e^{e^x} e^x \, dx$ is equal to
(a) $e^{e^x} e^x + C$ *(b)* $e^{e^x} + C$ *(c)* $2e^{e^x} e^x + C$ *(d)* $2e^{e^x} + C$

71. For three vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$, which of the following expressions is not equal to any of the remaining three?
(a) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$ *(b)* $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ *(c)* $(\mathbf{b} \times \mathbf{c}) \cdot \mathbf{a}$ *(d)* $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})$

72. If $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are three non-zero vectors such that each one of them being perpendicular to the sum of the other two vectors, then the value of $|\mathbf{p} + \mathbf{q} + \mathbf{r}|^2$ is
(a) $\frac{1}{2}(|\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2)$ *(b)* $2(|\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2)$
(c) $|\mathbf{p}| + |\mathbf{q}| + |\mathbf{r}|$ *(d)* $|\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2$

73. A line passing through $P(4, 2)$ meets the X and Y-axes at A and B, respectively. If O is the origin, then locus of the centre of the circumference of ΔAOB is
(a) $\frac{1}{x} + \frac{2}{y} = 1$ *(b)* $\frac{2}{x} + \frac{2}{y} = 1$ *(c)* $\frac{1}{x} + \frac{1}{y} = 2$ *(d)* $\frac{2}{x} + \frac{1}{y} = 1$

74. The probability that a man can hit a target is 3/4. He tries 5 times. The probability that he will hit the target atmost one time, is
(a) $\left(\frac{3}{4}\right)^5$ *(b)* $\left(\frac{1}{4}\right)^3$
(c) $\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3$ *(d)* None of these

75. A tower stands at the centre of a circular park. P and Q are two points on the boundary of the park such that PQ $(= a)$ subtends an angle of 60° at the foot of the tower and the angle of elevation of the top of the first tower from P or Q is 30°. The height of the tower is
(a) $\sqrt{3}$ *(b)* $\frac{a}{\sqrt{3}}$ *(c)* $2a\sqrt{3}$ *(d)* $\frac{2a}{\sqrt{3}}$

Answers with Explanation

1. *(c)* Given equation of ellipse is $4x^2 + 9y^2 = 36$ ...(i)

On differentiating w.r.t. x, we get

$$8x + 18y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-4x}{9y}$$

Since, the tangent is equally inclined to the axes.

$$\therefore \quad \frac{dy}{dx} = \pm 1 \Rightarrow \frac{-4x}{9y} = \pm 1 \Rightarrow y = \frac{-4}{9}x, \frac{4}{9}x$$

$$\therefore \quad 4x^2 + 9 \times \frac{16}{81}x^2 = 36$$

$$\Rightarrow \quad \frac{52}{9}x^2 = 36 \Rightarrow x = \pm\frac{9}{\sqrt{13}}$$

When $x = \frac{9}{\sqrt{13}}$, then $y = \frac{4}{\sqrt{13}}$

2. *(b)* Given, $z^2 + z + 1 = 0$

$$\therefore \quad z = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-1 \pm \sqrt{-3}}{2}$$

$$\Rightarrow \quad z = -\omega, \omega^2$$

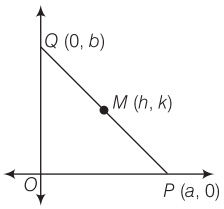
Now, $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \ldots + \left(z^6 + \frac{1}{z^6}\right)^2$

$= (-1)^2 + (-1)^2 + (1+1)^2 + (-1)^2 + (-1)^2 + (1+1)^2$

[for $z = \omega$ or $z = \omega^2$, we get same result]

$= 1 + 1 + 4 + 1 + 1 + 4 = 12$

3. *(a)* Let PQ be a rod of length l and $M(h,k)$ be a mid-point.



Let coordinates of P and Q be $(a, 0)$ and $(0, b)$, respectively.

$$\therefore \quad h = \frac{a+0}{2} \text{ and } k = \frac{0+b}{2}$$

$$\Rightarrow \quad h = \frac{a}{2} \text{ and } k = \frac{b}{2}$$

In ΔPOQ, we have

$$PQ^2 = OP^2 + OQ^2$$

$$\Rightarrow \quad a^2 + b^2 = l^2$$

$$\Rightarrow \quad (2h)^2 + (2k)^2 = l^2$$

$$\Rightarrow \quad 4h^2 + 4k^2 = l^2$$

Hence, the locus of the point (h,k) is $4x^2 + 4y^2 = l^2$.

4. *(b)* Let $L = \lim\limits_{x \to 0} \frac{(1 - \cos 2x)\sin 7x}{x^2 \sin 5x}$

$$= \lim_{x \to 0} \frac{2\sin^2 x}{x^2} \cdot \frac{\sin 7x}{7x} \cdot 7x \cdot \frac{5x}{\sin 5x} \cdot \frac{1}{5x}$$

$$= 2\lim_{x \to 0}\left(\frac{\sin x}{x}\right)^2 \cdot \lim_{x \to 0}\frac{\sin 7x}{7x} \cdot \lim_{x \to 0}\frac{5x}{\sin 5x} \cdot \frac{7}{5}$$

$$= 2(1) \cdot (1) \cdot (1) \cdot \frac{7}{5} = \frac{14}{5}$$

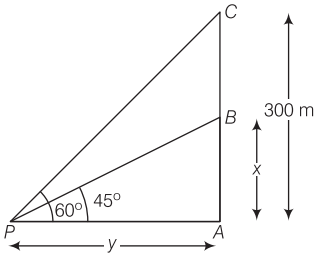
5. *(c)* Given, $\frac{x^2}{1-k} - \frac{y^2}{1+k} = 1$, where $|k| < 1$

$\Rightarrow$ $1 - k$ is positive and $1 + k$ is also positive.

$\therefore$ Given equation is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Hence, it represents a hyperbola when $|k| < 1$.

6. *(a)* Let height of the lower plane from the ground be x and $AP = y$.



Now, in ΔABP,

$$\tan 45° = \frac{AB}{PA} = \frac{x}{y} \Rightarrow 1 = \frac{x}{y} \Rightarrow x = y \quad ...(i)$$

and in ΔAPC, $\tan 60° = \frac{AC}{AP} = \frac{300}{y}$

$$\Rightarrow \quad \sqrt{3} = \frac{300}{y} \Rightarrow y = \frac{300}{\sqrt{3}} = 100\sqrt{3}$$

$$\therefore \quad x = 100\sqrt{3} \text{ m} \quad [\because x = y, \text{from Eq. (i)}]$$

7. *(b)* Let $\mathbf{p} = p_1\hat{\mathbf{i}} + p_2\hat{\mathbf{j}} + p_3\hat{\mathbf{k}}$

$$\therefore \quad |\mathbf{p} \times \hat{\mathbf{i}}| = |(p_1\hat{\mathbf{i}} + p_2\hat{\mathbf{j}} + p_3\hat{\mathbf{k}}) \times \hat{\mathbf{i}}| = |p_3\hat{\mathbf{j}} - p_2\hat{\mathbf{k}}|$$

Now, $|\mathbf{p} \times \hat{\mathbf{i}}|^2 = p_3^2 + p_2^2$

Similarly, $|\mathbf{p} \times \hat{\mathbf{j}}|^2 = p_3^2 + p_1^2$

and $|\mathbf{p} \times \hat{\mathbf{k}}|^2 = p_1^2 + p_2^2$

$$\therefore \quad |\mathbf{p} \times \hat{\mathbf{i}}|^2 + |\mathbf{p} \times \hat{\mathbf{j}}|^2 + |\mathbf{p} \times \hat{\mathbf{k}}|^2 = 2(p_1^2 + p_2^2 + p_3^2) = 2p^2$$

8. *(c)* Given, $f(x) = \frac{\tan\pi[(x - \pi)]}{1 + [x]^2}$

We know that, $\pi[(x - \pi)] = n\pi$ and $\tan n\pi = 0$.

$\because \quad 1 + [x^2] \neq 0$

$\therefore \quad f(x) = 0, \forall\, x$

So, $f(x)$ is a continuous function.

Thus, $f'(x)$, $f''(x)$, ... all exist for every x, their value being 0.

Hence, $f'(x)$ exists for all x.

9. *(b)* (a) $f(-x) = \frac{a^{-x} + 1}{a^{-x} - 1} = \frac{1 + a^x}{1 - a^{-x}} = -f(x)$

(b) $f(-x) = (-x)\frac{a^{-x} - 1}{a^{-x} + 1} = (-x)\frac{1 - a^x}{1 + a^x} = f(x)$

(c) $f(-x) = \log_2((-x) + \sqrt{(-x)^2 + 1})$

$= -\log_2(x + \sqrt{x^2 + 1}) = -f(x)$

(d) $f(-x) = \frac{a^{(-x)} + a^{-(-x)}}{a^{(-x)} - a^{-(-x)}} = \frac{a^{-x} + a^x}{a^{-x} - a^x} = -f(x)$

Here, only option (b) fulfill the condition for even function.

10. *(d)* Given, $\sin 5x \cos 3x = \sin 6x \cos 2x$

$\Rightarrow \quad 2\sin 5x \cos 3x = 2\sin 6x \cos 2x$

$\Rightarrow \quad \sin 8x + \sin 2x = \sin 8x + \sin 4x$

$\therefore \quad \sin 4x - \sin 2x = 0$

or $\quad 2\sin x \cos 3x = 0$

$\therefore \quad \sin x = 0$

$\Rightarrow \quad x = 0, \pi$

and $\quad \cos 3x = 0$

$\Rightarrow \quad x = \left(n + \frac{1}{2}\right)\frac{\pi}{3} = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

$\therefore$ Number of solutions = 5

11. *(c)* Given line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$.

Since, (2, 1, –2) lies on the plane.

$\therefore \quad 2\alpha + \beta = -5 \quad \ldots(i)$

Also, normal 1, 3, $-\alpha$ of the plane is perpendicular to the line $3 - 15 - 2\alpha = 0$.

$\therefore \quad \alpha = -6$ and $\beta = 7$ [from Eq. (i)]

Hence, $(\alpha, \beta) \equiv (-6, 7)$

12. *(d)* Given,

$$5^x + (2\sqrt{3})^{2x} \geq 13^x$$

$$\Rightarrow \quad \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x \geq 1 \quad \ldots(i)$$

Let $\quad \cos\alpha = \frac{5}{13}$

and $\quad \sin\alpha = \frac{12}{13}$

From Eq. (i), $\cos^x \alpha + \sin^x \alpha \geq 1$

Equality holds for $x = 2$

Both $\cos\alpha$ and $\sin\alpha$ increases for $x < 2$.

$\therefore \quad \cos^x \alpha + \sin^x \alpha \geq 1$, if $x < 2$

$\Rightarrow \quad x \leq 2$

Hence, $\quad x \in (-\infty, 2]$

13. *(a)* Let $I = \int \frac{\log(x/e)}{(\log x)^2}\, dx$

$$\Rightarrow \quad I = \int \frac{\log x - 1}{(\log x)^2}\, dx \qquad [\because \log_e e = 1]$$

Put $\quad t = \log x \Rightarrow x = e^t$

$\Rightarrow \quad dx = e^t dt$

$$\therefore \quad I = \int e^t \left(\frac{1}{t} - \frac{1}{t^2}\right) dx = \frac{e^t}{t} + C$$

$$[\because \int e^x [f(x) + f'(x)]\, dx = e^x f(x)]$$

$$= \frac{x}{\log x} + C$$

14. *(d)* Given, $A = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$

$\therefore \quad |A| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$

$\because \quad A = \text{adj}(P)$

$\therefore \quad |A| = |\text{adj } P| = |P|^2 = 16 \qquad [\because |P| = 4, \text{ given}]$

Now $\quad 2\alpha - 6 = 16$

$\Rightarrow \quad \alpha = 11$

15. *(c)* Let $\mathbf{A}_1 = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{A}_2 = 3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}}$

$\mathbf{A}_3 = \mathbf{r} = x\mathbf{p} + y\mathbf{q}$

$= x(\hat{\mathbf{i}} + \hat{\mathbf{j}}) + y(\hat{\mathbf{j}} + \hat{\mathbf{k}})$

$[\because \mathbf{p} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\mathbf{q} = \hat{\mathbf{j}} + \hat{\mathbf{k}}]$

$= x\hat{\mathbf{i}} + (x + y)\hat{\mathbf{j}} + y\hat{\mathbf{k}} = \mathbf{r}$

$\because \quad \mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$ are coplanar.

$$\therefore \quad \begin{vmatrix} 1 & -2 & 1 \\ 3 & 2 & -1 \\ x & x+y & y \end{vmatrix} = 0$$

On applying $C_2 \to C_2 - (C_1 + C_3)$, we get

$$\begin{vmatrix} 1 & -4 & 1 \\ 3 & 0 & -1 \\ x & 0 & y \end{vmatrix} = 0$$

$$\Rightarrow \quad 4(3y + x) = 0$$

$$\therefore \quad \frac{x}{y} = -3$$

16. *(d)*

q	p	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	T	F	T

$\therefore$ Statement $p \to (q \to p)$ is equivalent to $p \to (p \vee q)$.

17. *(d)* Required number of ways

= Coefficient of x^{10} in $(1 + x + x^2 + \ldots)^4$

= Coefficient of x^{10} in $(1 - x)^{-4}$

= Coefficient of x^{10} in $(1 + {}^4C_1 x + {}^5C_2 x^2 + {}^6C_3 x^3 + \ldots)$

$= {}^{13}C_3 = 286$

18. *(a)* We have, $y^2 = 4x \quad \ldots(i)$

and $\quad x^2 = 4ay \quad \ldots(ii)$

Any tangent to parabola (i) is

$$y = mx + \frac{1}{m} \qquad [\because a = 1] \quad \ldots(iii)$$

and any normal to parabola (ii) is

$$x = My - 2aM - aM^3 \quad \ldots(iv)$$

or $\quad y = \frac{1}{M}x + 2a + aM^2 \quad \ldots(v)$

If Eqs. (iii) and (v) are same, then

$$m = \frac{1}{M} \text{ and } 2a + aM^2 = \frac{1}{m} = M$$

$$\Rightarrow \quad aM^2 - M + 2a = 0$$

For above equation, two real roots are positive, if

$$1 - 8a^2 > 0$$

$$\Rightarrow \quad a^2 < \frac{1}{8}$$

$$\therefore \quad |a| < \frac{1}{2\sqrt{2}}$$

19. *(c)* Given, $f(x) = \cos x + \frac{1}{2}\cos 2x - \frac{1}{3}\cos 3x$

Here, $f(x)$ is periodic with period 2π.

Thus, difference between the greatest and the least values of the function $f(x)$ is equal to difference between the greatest and the least values on the interval $[0, 2\pi]$.

Now, $f'(x) = -\sin x - \sin 2x + \sin 3x$

$$= -(\sin x + \sin 2x) + \sin 3x$$

$$= -2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{x}{2}\right) + 2\sin\left(\frac{3x}{2}\right)\cos\left(\frac{3x}{2}\right)$$

$$= -2\sin\left(\frac{3x}{2}\right)\left[\cos\left(\frac{x}{2}\right) - \cos\left(\frac{3x}{2}\right)\right]$$

$$= -2\sin\left(\frac{3x}{2}\right)\cdot 2\sin x \sin\left(\frac{x}{2}\right)$$

$$= -4\sin x \sin\left(\frac{x}{2}\right)\sin\left(\frac{3x}{2}\right)$$

On putting $f'(x) = 0$, we get

$$x = 0, \frac{2\pi}{3}, \pi, 2\pi$$

$\therefore$ $f(0) = \frac{7}{6}, f\left(\frac{2\pi}{3}\right) = \frac{-13}{2}$,

$f(\pi) = \frac{-1}{6}$ and $f(2\pi) = \frac{7}{6}$

Here, greatest and least values are $\frac{7}{6}$ and $\frac{-13}{12}$, respectively.

$\therefore$ Difference between greatest and least values of

$$f(x) = \frac{7}{6} - \left(\frac{-13}{12}\right) = \frac{9}{4}$$

20. *(c)* Let $\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_5 & {}^{11}C_k \\ {}^{11}C_6 & {}^{11}C_7 & {}^{12}C_{k+2} \\ {}^{12}C_8 & {}^{12}C_9 & {}^{13}C_{k+4} \end{vmatrix} = 0$

On applying $C_2 \to C_1 + C_2$, we get

$$\Delta = \begin{vmatrix} {}^{10}C_4 & {}^{10}C_4 + {}^{10}C_5 & {}^{11}C_k \\ {}^{11}C_6 & {}^{11}C_6 + {}^{11}C_7 & {}^{12}C_{k+2} \\ {}^{12}C_8 & {}^{12}C_8 + {}^{12}C_9 & {}^{13}C_{k+4} \end{vmatrix} = 0$$

$$= \begin{vmatrix} {}^{10}C_4 & {}^{11}C_5 & {}^{11}C_k \\ {}^{11}C_6 & {}^{12}C_7 & {}^{12}C_{k+2} \\ {}^{12}C_8 & {}^{13}C_9 & {}^{13}C_{k+4} \end{vmatrix} = 0 \qquad [\because {}^{n}C_r + {}^{n}C_{r+1} = {}^{n+1}C_r]$$

$\because$ C_2, C_3 will be identical for $k = 5$.

$\therefore$ $k = 5$ satisfies the above result.

21. *(c)* Let $x_1, x_2, \ldots, x_n$ be positive numbers.

According to the question, $x_1x_2x_3 \ldots x_n = 1$

$\because$ AM $\geq$ GM

$$\therefore \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) \geq (x_1x_2 \ldots x_n)^{1/2}$$

$$\Rightarrow \left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) \geq 1 \qquad [\because x_1x_2 \ldots x_n = 1]$$

$\therefore x_1 + x_2 + \ldots + x_n \geq n$

So, $x_1 + x_2 + \ldots + x_n$ can never be less than n.

22. *(a)* Given, in a binomial expansion of $(a - b)^n$, $n \geq 5$, the sum of 5th and 6th terms is equal to zero.

$\therefore$ ${}^{n}C_4a^{n-4}(-b)^4 + {}^{n}C_5a^{n-5}(-b)^5 = 0$

$$\Rightarrow \frac{n!}{(n-4)!4!}\cdot a^{n-4}\cdot b^4 - \frac{n!}{(n-5)!5!}\cdot a^{n-5}\cdot b^5 = 0$$

$$\Rightarrow \frac{n!}{(n-5)!4!}\cdot a^{n-5}\cdot b^4\left(\frac{a}{n-4} - \frac{b}{5}\right) = 0$$

$$\Rightarrow \frac{a}{b} = \frac{n-4}{5}$$

23. *(b)* Let $L = \lim_{n\to\infty}\left(\frac{a - 1 + \sqrt[n]{b}}{a}\right)^n = \lim_{n\to\infty}\left\{1 + \left(\frac{\sqrt[n]{b} - 1}{a}\right)\right\}^n$

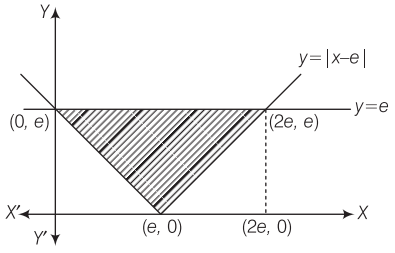
$$= e^{\lim_{n\to\infty} n\left(\frac{\sqrt[n]{b} - 1}{a}\right)} = e^{\frac{1}{a}\lim_{n\to\infty}\frac{b^{1/n} - 1}{1/n}}$$

$$= e^{\frac{1}{a}\ln b} = e^{\ln(b)^{1/a}} = b^{1/a} = \sqrt[a]{b}$$

24. *(a)* We have, $y = x^{\frac{1}{\ln x}}$

$$\Rightarrow \ln y = \frac{1}{\ln x}\cdot \ln x = 1, \ln x \neq 0$$

$$\Rightarrow y = e, x \neq 0, x > 0$$



$\therefore$ Required area $= \frac{1}{2} \times 2e \times e = e^2$ sq units

25. *(c)* Probability that the electronic component fails when first use,

$$P(F) = 0.10$$

$\therefore$ $P(F') = 1 - P(F) = 1 - 0.10 = 0.90$

Let E be the event that a new component will last for one year, then

$$P(E) = P(F)P\left(\frac{E}{F}\right) + P(F')P\left(\frac{E}{F'}\right)$$

[by using total probability theorem]

$$= 0.10 \times 0 + 0.90 \times 0.99 = 0.891$$

26. *(a)* Let coordinates of point P be (h,k) and coplanar points be $(0, 0)$ and $(a, 0)$.

$\because$ Distance of point P from point $(0, 0)$

$$= \sqrt{h^2 + k^2}$$

and distance of point P from point $(a, 0)$

$$= \sqrt{(h-a)^2 + k^2}$$

According to the question,

$$\frac{\sqrt{h^2 + k^2}}{\sqrt{(h-a)^2 + k^2}} = k \qquad [\because k \neq 1]$$

where k is any number.

$$\Rightarrow \quad h^2 + k^2 + \left(\frac{k^2}{k^2-1}\right)(a^2 - 2ah) = 0$$

$\therefore$ Required equation of locus is

$$x^2 + y^2 + \left(\frac{k^2}{k^2-1}\right)(a^2 - 2ax) = 0$$

which is equation of circle.

27. ***(d)*** Let $\mathbf{OA} = 2\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{OB} = \hat{\mathbf{i}} - 3\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$

and $\mathbf{OC} = 3\hat{\mathbf{i}} - 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$

Here, $a = |\mathbf{OA}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$b = |\mathbf{OB}| = \sqrt{(1)^2 + (-3)^2 + (-5)^2} = \sqrt{35}$

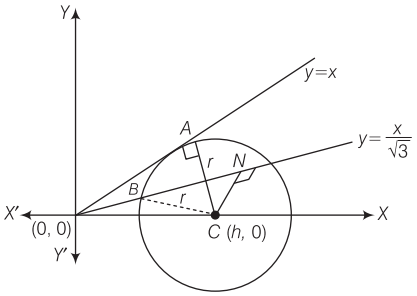
and $c = |\mathbf{OC}| = \sqrt{(3)^2 + (-4)^2 + (-4)^2} = \sqrt{41}$

$$\therefore \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(\sqrt{35})^2 + (\sqrt{41})^2 - (\sqrt{6})^2}{2\cdot\sqrt{35}\cdot\sqrt{41}}$$

$$= \frac{35 + 41 - 6}{2\cdot\sqrt{35}\cdot\sqrt{41}} = \frac{70}{2\cdot\sqrt{35}\cdot\sqrt{41}} = \sqrt{\frac{35}{41}}$$

$$\therefore \quad \cos^2 A = \frac{35}{41}$$

28. ***(b)*** Let coordinates of C be $(h, 0)$ and radius be r.



Since, CA is perpendicular to the tangent.

$$\therefore \quad r = \frac{|h - 0|}{\sqrt{1^2 + 1^2}}$$

$$\Rightarrow \quad h = \sqrt{2}r \qquad \ldots(i)$$

Also, CN is perpendicular to the chord of line $y = \frac{x}{\sqrt{3}}$.

$$\therefore \quad CN = \frac{\left|\frac{h}{\sqrt{3}} - 0\right|}{\sqrt{\frac{1}{3} + 1}} = \frac{h}{2}$$

In ΔBNC, $r^2 = 1^2 + \left(\frac{h}{2}\right)^2 = 1 + \frac{h^2}{4}$ $\qquad \ldots(ii)$

From Eqs. (i) and (ii), $\quad r^2 = 1 + \frac{2r^2}{4}$

$$\Rightarrow \quad r = \sqrt{2}$$

29. ***(a)*** Given, $\quad |z - 1| = |z + i|$

$$\Rightarrow \quad |(x + iy) - 1| = |(x + iy) + i|$$

$$\Rightarrow \quad |(x - 1) + iy| = |x + i(y + 1)|$$

$$\Rightarrow \quad (x - 1)^2 + y^2 = x^2 + (y + 1)^2$$

$$\Rightarrow \quad x^2 + 1 - 2x + y^2 = x^2 + y^2 + 1 + 2y$$

$$\Rightarrow \quad x + y = 0$$

which is a line through the origin.

30. ***(a)*** Given equation of curve is

$$y = \frac{8}{(4 + x^2)} \qquad \ldots(i)$$

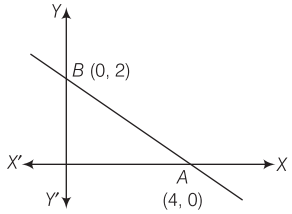
At $x = 2$, $\quad y = \frac{8}{4 + (2)^2} = 1$

$\therefore$ Slope of the tangent to curve (i),

$$m = \frac{dy}{dx} = \frac{-16x}{(4 + x^2)^2}$$

and slope of the tangent at point (2, 1),

$$m = \left(\frac{dy}{dx}\right)_{(2,1)} = \frac{-16(2)}{[4 + (2)^2]^2} \Rightarrow m = -\frac{1}{2}$$



Now, the equation of the tangent is

$$y - 1 = \frac{-1}{2}(x - 2)$$

$$\Rightarrow \quad x + 2y = 4$$

$$\Rightarrow \quad \frac{x}{4} + \frac{y}{2} = 1$$

Here, AB is the tangent to the curve (i).

$\therefore$ Required area of $\Delta OAB = \frac{1}{2} \times 4 \times 2 = 4$ sq units

31. ***(c)*** Given, $\quad \mathbf{a} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$

and $\quad \mathbf{b} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$

Let $\quad \mathbf{c} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$\because \quad \mathbf{a} \cdot \mathbf{c} = 4$

$\therefore (2)(x) + (1)(y) + (-1)(z) = 4 \quad \Rightarrow 2x + y - z = 4$

and $\mathbf{a} \times \mathbf{c} = \mathbf{b}$

$$\therefore \quad \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & 1 & -1 \\ x & y & z \end{vmatrix} = \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

$$\Rightarrow \hat{\mathbf{i}}(z + y) - \hat{\mathbf{j}}(2z + x) + \hat{\mathbf{k}}(2y - x) = \hat{\mathbf{j}} + \hat{\mathbf{k}}$$

On comparing coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ both sides, we get

$$z + y = 0,\ 2z + x = -1 \text{ and } 2y - x = 1$$

On solving these equations, we get

$$x = 1,\ y = 1 \text{ and } z = -1$$

$$\therefore \quad \mathbf{c} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$$

32. ***(c)*** Given series is

$$\log a + \log \frac{a^2}{b} + \log \frac{a^3}{b^2} + \ldots \text{ upto } n \text{ terms}$$

Here, $T_n - T_{n-1} = \log \frac{a^n}{b^{n-1}} - \log \frac{a^{n-1}}{b^{n-2}}$

$$= \log \frac{a^n}{a^{n-1}} \cdot \frac{b^{n-2}}{b^{n-1}}$$

$$= \log \frac{a}{b} = \text{Constant, as it is independent of } n.$$

$\therefore$ The given series is an AP whose first term = $\log a$ and common difference = $\log \frac{a}{b}$

$\therefore$ Sum of the series

$$S = \frac{n}{2}\left[2\log a + (n-1)\log\frac{a}{b}\right]$$
$$= \frac{n}{2}\left[n\log\frac{a}{b} + 2\log a - (\log a - \log b)\right]$$
$$= \frac{n}{2}\left(n\log\frac{a}{b} + \log ab\right)$$

33. ***(c)*** Let the equation of circle be

$$x^2 + y^2 + 2gx + 2fy + c = 0 \quad \ldots(i)$$

Since, circle (i) passes through the point (a, b).

$$\therefore \quad a^2 + b^2 + 2g(a) + 2f(b) + c = 0 \quad \ldots(ii)$$

$\because$ Circle (i) cuts $x^2 + y^2 = k^2$ orthogonally.

$$\therefore \quad 2g(0) + 2f(0) = c - k^2$$
$$\Rightarrow \quad c = k^2$$

On putting $c = k^2$ in Eq. (ii), we get

$$2ag + 2bf + (a^2 + b^2 + k^2) = 0$$

$\therefore$ The locus of the centre $(-g, -f)$ will be

$$-2ax - 2by + (a^2 + b^2 + k^2) = 0$$
$$\Rightarrow \quad 2ax + 2by - (a^2 + b^2 + k^2) = 0$$

34. ***(a)*** We have, $|2\hat{\mathbf{a}} \times 3\hat{\mathbf{b}}| = 1$

$$\Rightarrow \quad 6|\hat{\mathbf{a}}||\hat{\mathbf{b}}||\sin\theta| = 1$$
$$\Rightarrow \quad \sin\theta = \frac{1}{6} \qquad [\because |\hat{\mathbf{a}}||\hat{\mathbf{b}}| = 1]$$

Since, θ is an acute angle, therefore there is exactly one value of θ for which $(2\hat{\mathbf{a}} \times 3\hat{\mathbf{b}})$ is a unit vector.

35. ***(d)*** Given, $\mathbf{a} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

$\mathbf{b} = \hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}$ and $\mathbf{c} = 7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}}$

Let $\mathbf{P} = \mathbf{a} + \mathbf{b} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}) + (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$

and $\mathbf{Q} = \mathbf{b} + \mathbf{c} = (\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 5\hat{\mathbf{k}}) + (7\hat{\mathbf{i}} + 9\hat{\mathbf{j}} + 11\hat{\mathbf{k}})$

$$= 8\hat{\mathbf{i}} + 12\hat{\mathbf{j}} + 16\hat{\mathbf{k}}$$

$$\therefore \text{ Area of parallelogram} = \frac{1}{2}|\mathbf{P} \times \mathbf{Q}| = \frac{1}{2}\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & 4 & 6\\ 8 & 12 & 16\end{vmatrix}$$

$$= \frac{1}{2}|\hat{\mathbf{i}}(4\times16 - 12\times6) - \hat{\mathbf{j}}(2\times16 - 8\times6) + \hat{\mathbf{k}}(2\times12 - 8\times4)|$$
$$= \frac{1}{2}|-8\hat{\mathbf{i}} + 16\hat{\mathbf{j}} - 8\hat{\mathbf{k}}| = |-4\hat{\mathbf{i}} + 8\hat{\mathbf{j}} - 4\hat{\mathbf{k}}|$$
$$= \sqrt{(-4)^2 + (8)^2 + (-4)^2} = 4\sqrt{6} \text{ sq units}$$

36. ***(a)*** Let $\Delta = \begin{vmatrix}1 & \log_x y & \log_x z\\ \log_y x & 1 & \log_y z\\ \log_z x & \log_z y & 1\end{vmatrix} = \begin{vmatrix}1 & \frac{\log y}{\log x} & \frac{\log z}{\log x}\\ \frac{\log x}{\log y} & 1 & \frac{\log z}{\log y}\\ \frac{\log x}{\log z} & \frac{\log y}{\log z} & 1\end{vmatrix}$

On dividing and multiplying R_1, R_2, R_3 by $\log x$, $\log y$, $\log z$ respectively, we get

$$\Delta = \frac{1}{\log x \log y \log z}\begin{vmatrix}\log x & \log y & \log z\\ \log x & \log y & \log z\\ \log x & \log y & \log z\end{vmatrix} = 0$$

37. ***(b)*** Given, $f'(\sin x) = \log_e x = \log_e[\pi - \sin^{-1}(\sin x)]$

$$\left[\because \sin^{-1}(\sin x) = \pi - x, \text{ for } x \in \left(\frac{\pi}{2}, \pi\right)\right]$$

Let $\sin x = z$

Then, $\quad f'(z) = \log_e(\pi - \sin^{-1} z) \quad \ldots(i)$

Also, given that $y = f(a^x)$

$$\Rightarrow \quad \frac{dy}{dx} = a^x \log_e a\, f'(a^x)$$
$$= a^x \log_e a \log_e(\pi - \sin^{-1} a^x) \text{ [from Eq. (i)]}$$

38. ***(b)*** We have, $x^2 + 5 = 2x - 4\cos(a + b)$

$$\Rightarrow \quad x^2 - 2x + 1 + 4 = -4\cos(a + b)$$
$$\Rightarrow \quad (x-1)^2 + 4 = -4\cos(a + b)$$
$$\Rightarrow \quad (x-1)^2 + 4[1 + \cos(a + b)] = 0$$
$$\Rightarrow \quad x = 1 \text{ and } 1 + \cos(a+b) = 0$$
$$\Rightarrow \quad \cos(a + b) = -1$$
$$\Rightarrow \quad a + b = \pi, 3\pi, \ldots$$

Hence, maximum value of $a + b$ in $[0, 2\pi]$ is π.

39. ***(d)*** Given, $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n} \quad \ldots(i)$

$\because$ Line (i) is parallel to XY-plane.

So, normal line to plane is perpendicular to the Z-axis.

$\therefore$ Direction ratios of z-coordinate are zero i.e. $n = 0$.

40. ***(d)*** We know that, by Poisson distribution,

$$P(X = r) = \frac{e^{-m}m^r}{r!}$$

$$\therefore P(X \le 1) = P(X = 0) + P(X = 1) = e^{-m} + \frac{e^{-m}m}{1!}$$

Given, m = mean = 5

$$\therefore \quad P(X \le 1) = e^{-5} + 5 \times e^{-5}$$
$$= e^{-5}(1 + 5) = \frac{6}{e^5}$$

41. ***(a)*** Let t be the period of $\sin x + \cos ax$.

$\therefore \sin(t + x) + \cos a(t + x) = \sin x + \cos ax$ for all x.

On putting $x = 0$ and $x = -t$, we get

$\sin t + \cos at = 1$ and $1 = -\sin t + \cos at$

On solving above equations, we get

$$\sin t = 0 \text{ and } \cos at = 1$$

$\therefore \lambda = n\lambda$ and $at = 2m\pi$

where, m and n are non-zero integers.

Here, $\quad \frac{a\lambda}{\lambda} = \frac{2m\pi}{n\pi} \Rightarrow a = \frac{2m}{n} \qquad [\because \lambda \neq 0]$

which is a rational number.

42. ***(a)*** We have, $(99)^{50} = (100 - 1)^{50}$

$$= (100)^{50} - {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} - \ldots \quad \ldots(i)$$

and $(101)^{50} = (100 + 1)^{50}$

$$= (100)^{50} + {}^{50}C_1(100)^{49} + {}^{50}C_2(100)^{48} + \ldots \quad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$$(101)^{50} - (99)^{50} = 2\{{}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47} + \ldots\}$$
$$= 2 \times {}^{50}C_1(100)^{49} + 2 \times \{{}^{50}C_3(100)^{47} + \ldots\}$$
$$= \{100\,(100)^{49} + \text{positive number}\} > (100)^{50}$$

$\therefore (101)^{50} > (100)^{50} + (99)^{50}$

43. ***(b)*** Given, $4x^2 - 9y^2 - 36 = 0$

$$\Rightarrow \quad \frac{x^2}{9} - \frac{y^2}{4} = 1 \qquad \ldots(i)$$

Then, the equation of the chord of contact of tangents from (x_1, y_1) and (x_2, y_2) to the given hyperbola are

$$\frac{xx_1}{9} - \frac{yy_1}{4} = 1 \qquad \ldots(ii)$$

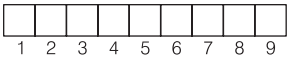
and $$\frac{xx_2}{9} - \frac{yy_2}{4} = 1 \qquad \ldots(iii)$$

$\because$ Lines (ii) and (iii) are at right angles.

$$\therefore \quad \frac{4}{9} \times \frac{x_1}{y_1} \times \frac{4}{9} \times \frac{x_2}{y_2} = -1$$

$$\Rightarrow \quad \frac{x_1 x_2}{y_1 y_2} = \frac{-81}{16}$$

44. ***(b)*** According to the question, while forming the numbers, odd digit occupy even positions.



The four digits 5, 5, 7 and 7 can be arranged in box 2, 4, 6 and 8 in $\frac{4!}{2!\,2!}$ i.e. 6 ways.

The five digits 4, 4, 6, 6 and 6 can be arranged in box 1, 3, 5, 7 and 9 in $\frac{5!}{2!\,3!}$ i.e. 10 ways.

$\therefore$ Total number of arrangements $= 6 \times 10$

$= 60$ ways

45. ***(b)*** Let $S_{99} = 2 + 7 + 14 + 23 + 34 + \ldots + t_{98} + t_{99}$ $\ldots(i)$

Also, $S_{99} = \quad 2 + 7 + 14 + 23 + \ldots + t_{98} + t_{99}$ $\ldots(ii)$

On subtracting Eq. (ii) from Eq. (i), we get

$$0 = 2 + (5 + 7 + 9 + 11 + \ldots \text{upto 98 terms}) - t_{99}$$

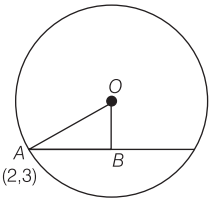
$$\therefore \quad t_{99} = 2 + (5 + 7 + 9 + 11 + \ldots \text{upto 98 terms})$$

$$= 2 + \frac{98}{2}[2(5) + (98 - 1)2]$$

$$= 2 + 49(10 + 194)$$

$$= 2 + 49 \times 204 = 9998$$

46. ***(b)*** Let $A(2, 3)$ be a given point.



Here, B is the middle point of a chord of the circle $x^2 + y^2 = r^2$ through A.

$\therefore (OB)^2 = (OA)^2 - (AB)^2$

which is maximum when AB is minimum, i.e. when A coincides with B.

$\therefore$ Equation of the chord is $T = S_1$

i.e. $\quad 2x + 3y - r^2 = (2)^2 + (3)^2 - r^2$

$$\Rightarrow \quad 2x + 3y = 13$$

47. ***(d)*** Given, $\quad 3x + 4y = 9 \qquad \ldots(i)$

and $\quad y = mx + 1 \qquad \ldots(ii)$

On solving Eqs. (i) and (ii), we get

$$3x + 4(mx + 1) = 9$$

$$\therefore \quad x = \frac{5}{3 + 4m}$$

which is integer, if $3 + 4m = \pm 1$ or $3 + 4m = \pm 5$

$$\therefore \quad m = -1 \text{ or } m = -2$$

For two values of m, x-coordinate of M is also an integer.

48. ***(b)*** Let $L = \lim\limits_{x \to 0} \frac{1 - \cos^3 x}{x \sin x \cos x}$

$$= \lim_{x \to 0} \frac{(1 - \cos x)(1 + \cos^2 x + \cos x)}{x \sin x \cos x}$$

$$[\because a^3 - b^3 = (a - b)(a^2 + b^2 + ab)]$$

$$= \lim_{x \to 0} \frac{2\sin^2\left(\frac{x}{2}\right)(1 + \cos^2 x + \cos x)}{2x \sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)\cos x}$$

$$= \lim_{x \to 0} \frac{1}{2} \cdot \frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)} \cdot \frac{1 + \cos^2 x + \cos x}{\cos\left(\frac{x}{2}\right)\cos x} = \frac{1}{2} \times 1 \times \frac{3}{1} = \frac{3}{2}$$

49. ***(b)*** Let $P(h, k)$ be any point on $y^2 = 4x$ and its image about the line $x - y + 1 = 0$ is $Q(\alpha, \beta)$.

$\therefore$ Mid-point of P and Q lies on the line.

$$\Rightarrow \quad \frac{h + \alpha}{2} - \frac{k + \beta}{2} + 1 = 0 \Rightarrow h + \alpha - k - \beta + 2 = 0$$

$$\Rightarrow \quad h - k = -\alpha + \beta - 2 \qquad \ldots(i)$$

Also, $\quad \frac{\beta - k}{\alpha - h} \times 1 = -1 \qquad [\because m_1 m_2 = -1]$

$$\Rightarrow \quad \beta - k = -\alpha + h$$

$$\Rightarrow \quad h + k = \alpha + \beta \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

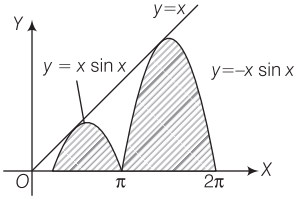
$$h = \beta - 1 \text{ and } k = \alpha + 1$$

$\because \quad k^2 = 4h \Rightarrow (\alpha + 1)^2 = 4(\beta - 1)$

Hence, the required curve is $(x + 1)^2 = 4(y - 1)$.

50. ***(d)*** Given, $y = x|\sin x|$

$$\Rightarrow \quad y = \begin{cases} x \sin x, & 0 \le x \le \pi \\ -x \sin x, & \pi \le x \le 2\pi \end{cases}$$



$$\therefore \text{ Required area} = \int_0^{\pi} x \sin x \, dx + \int_{\pi}^{2\pi} (-x \sin x) dx$$

$$= [x(-\cos x)]_0^{\pi} - \int_0^{\pi} (-\cos x) dx$$

$$- [x(-\cos x)]_{\pi}^{2\pi} + \int_{\pi}^{2\pi} (-\cos x) dx$$

$$= \pi + [\sin x]_0^{\pi} + (2\pi + \pi) - [\sin x]_{\pi}^{2\pi}$$

$$= 4\pi \text{ sq units}$$

51. ***(d)*** Let $L = \lim_{n\to\infty}\frac{1-(10)^n}{1+(10)^{n+1}} = \lim_{n\to\infty}\frac{10^n}{10^{n+1}}\left(\frac{\frac{1}{10^n}-1}{\frac{1}{10^{n+1}}+1}\right)$

$\Rightarrow \quad \frac{-\alpha}{10} = \frac{1}{10}\left(\frac{0-1}{0+1}\right) = \frac{-1}{10}$

$\therefore \quad \alpha = 1$

52. ***(a)*** Let p : 2 is prime and q : 3 is odd.

Given, $p \to q$

Negation of $p \to q$ is $\sim(p \to q) \Rightarrow p \wedge \sim q$

$\therefore$ 2 is prime and 3 is not odd.

53. ***(a)*** Let p : I become a teacher

and q : I will open a school.

Negation of $p \to q$ is $\sim(\sim p \vee q) \equiv p \wedge \sim q$

i.e. I will become a teacher and I will not open a school.

54. ***(c)*** By given condition, $2\cdot {}^mC_2 = {}^mC_1 + {}^mC_3$

$\Rightarrow \quad m(m-1) = m + \frac{m(m-1)(m-2)}{1\cdot 2\cdot 3}$

$\Rightarrow \quad m^2 - 9m + 14 = 0$

$\Rightarrow \quad (m-2)(m-7) = 0 \Rightarrow m = 2, 7$

Since, 6th term is 21.

$\therefore \quad T_6 = 21 = {}^7C_5[\sqrt{2^{\log(10-3^x)}}]^{7-5} \times \sqrt[5]{2^{(x-2)\log 3}}]^5$

$\Rightarrow \quad 21 = \frac{7\times 6}{2\times 1}\,2^{\log(10-3^x)}\cdot 2^{(x-2)\log 3}$

$\Rightarrow \quad 2^{\log(10-3^x)+(x-2)\log 3} = 1 = 2^0$

$\Rightarrow \quad \log(10-3^x) = (2-x)\log 3$

$\Rightarrow \quad \log(10-3^x) = \log 3^{(2-x)}$

$\Rightarrow \quad 10 - 3^x = 3^{(2-x)}$

$\Rightarrow \quad 3^x\cdot 10 - 3^{2x} = 9$

$\Rightarrow \quad 3^{2x} - 10\cdot 3^x + 9 = 0$

On putting $t = 3^x$, we get

$t^2 - 10t + 9 = 0$

$\Rightarrow \quad t^2 - t - 9t + 9 = 0$

$\Rightarrow \quad t(t-1) - 9(t-1) = 0$

$\Rightarrow \quad (t-1)(t-9) = 0$

$\Rightarrow \quad t = 1 \text{ or } 9$

$\therefore \quad 3^x = 1 \text{ or } 3^x = 9 \Rightarrow x = 0 \text{ or } x = 2$

55. ***(c)*** $\because (x^2 - 3x + 2) = (x-1)(x-2)$

$\therefore (x^2 - 3x + 2) > 0$, when $x < 1$ or $x > 2$

and $(x^2 - 3x + 2) < 0$, when $1 < x < 2$

Also, $\cos|x| = \cos x \quad [\because \cos(-x) = \cos x]$

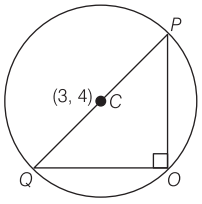
$\therefore f(x) = -(x^2-1)(x^2-3x+2) + \cos x, 1 \le x \le 2$

$= (x^2-1)(x^2-3x+2) + \cos x, x > 2$

Hence, $f(x)$ is not differentiable at $x = 2$ as LHD $\neq$ RHD.

56. ***(a)*** Given equation of circle is

$(x-3)^2 + (y-4)^2 = 25 \quad \ldots(i)$



Centre $\equiv (3, 4)$

and given equation of straight line is

$4x + 3y = 24 \quad \ldots(ii)$

$\because$ Line (ii) passes through centre (3, 4) of circle (i).

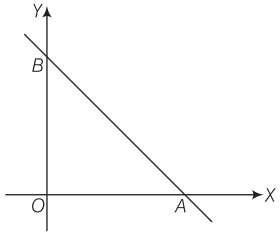
$\therefore$ The line (ii) is a diameter at circle (i).

$\because$ $O(0, 0)$ lies on the circle (i).

$\therefore \quad \angle POQ = \frac{\pi}{2} \quad \left[\because \text{angle in a semicircle is } \frac{\pi}{2}\right]$

$\therefore \quad OP \perp OQ$

57. ***(a)*** Given curve is $\sqrt{x} + \sqrt{y} = \sqrt{a} \quad \ldots(i)$



On differentiating both sides, we get

$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}}\cdot\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}} \quad \ldots(ii)$

$\therefore$ Equation of tangent at (x, y) is

$Y - y = \frac{dy}{dx}(X - x)$

$\Rightarrow \quad Y - y = -\frac{\sqrt{y}}{\sqrt{x}}(X - x) \quad$ [from Eq. (ii)]

$\Rightarrow \quad \frac{Y}{\sqrt{y}} - \sqrt{y} = -\frac{X}{\sqrt{x}} + \sqrt{x}$

$\Rightarrow \quad \frac{X}{\sqrt{x}} + \frac{Y}{\sqrt{y}} = \sqrt{x} + \sqrt{y} = \sqrt{a} \quad$ [from Eq. (i)]

$\Rightarrow \quad \frac{X}{\sqrt{a}\sqrt{x}} + \frac{Y}{\sqrt{a}\sqrt{y}} = 1$

According to the question,

$OA = \sqrt{a}\sqrt{x}$ and $OB = \sqrt{a}\sqrt{y}$

$\therefore \quad OA + OB = \sqrt{a}\sqrt{x} + \sqrt{a}\sqrt{y} = \sqrt{a}(\sqrt{x} + \sqrt{y})$

$= \sqrt{a}\cdot\sqrt{a} \quad$ [from Eq. (i)]

$= a$

58. ***(b)*** $\because x^2 + 2x + 3 = x^2 + 2x + 1 + 2 = (x+1)^2 + 2 \ge 2$

$\therefore \quad a = 2$ and $b = \lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2} = \frac{1}{2}$

$\Rightarrow \sum_{r=0}^{n} a^r\cdot b^{n-r} = b^n\sum_{r=0}^{n}\left(\frac{a}{b}\right)^r$

$= b^n\left\{1 + \left(\frac{a}{b}\right) + \left(\frac{a}{b}\right)^2 + \ldots + \left(\frac{a}{b}\right)^n\right\} = b^n\left\{\frac{1-\left(\frac{a}{b}\right)^{n+1}}{1-\frac{a}{b}}\right\}$

$= \frac{b^{n+1} - a^{n+1}}{b-a} = \frac{\left(\frac{1}{2}\right)^{n+1} - 2^{n+1}}{\left(\frac{1}{2} - 2\right)} = \frac{4^{n+1} - 1}{3\cdot 2^n}$

59. ***(b)*** Given equations of lines are

$$\frac{x-1}{3}=\frac{y+2}{2}=\frac{z}{-4} \quad \ldots(i)$$

and $$\frac{x}{2}=\frac{y-1}{-3}=\frac{z-2}{2} \quad \ldots(ii)$$

Now, equation of plane passing through $(2, -1, -3)$ is

$$a(x-2)+b(y+1)+c(z+3)=0 \quad \ldots(iii)$$

$\because$ Lines (i) and (ii) are parallel to plane (iii).

$\therefore$ $$3a+2b-4c=0 \quad \ldots(iv)$$

and $$2a-3b+2c=0 \quad \ldots(v)$$

On eliminating a, b and c from Eqs. (iii), (iv) and (v), we get

$$\begin{vmatrix} x-2 & y+1 & z+3 \\ 3 & 2 & -4 \\ 2 & -3 & 2 \end{vmatrix}=0$$

$\Rightarrow (x-2)(4-12)-(y+1)(6+8)+(z+3)(-9-4)=0$

$\Rightarrow 8x+14y+13z+37=0$

60. ***(a)*** The number of injections from $\{m_1, m_2, m_3, m_4\}$ to $\{n_1, n_2, n_3, n_4, n_5, n_6, n_7\}$

$$= {}^7P_4 = \frac{7!}{(7-4)!}$$

$$= \frac{7!}{3!}=\frac{7\times 6\times 5\times 4\times 3!}{3!}=840$$

61. ***(c)*** Let $$y=\left[0.16^{\log_{0.25}\left(\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty\right)}\right]^2$$

$$=\left[0.16^{\log_{0.25}\left(\frac{1/3}{1-1/3}\right)}\right]^2$$

$$=[0.16^{\log_{(0.5)^2}0.5}]^2$$

$$=[0.16^{\frac{1}{2}\log_{0.5}0.5}]^2=(0.16^{1/2})^2$$

$$=(0.16)^1=0.16$$

62. ***(a)*** Given, $y_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$

Here, $y_1=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

and $y_2=\cos\frac{\pi}{2^2}+i\sin\frac{\pi}{2^2}$

$\therefore y_1\cdot y_2\cdot y_3\ldots$

$$=\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\times\left(\cos\frac{\pi}{2^2}+i\sin\frac{\pi}{2^2}\right)\times\ldots$$

$$=\cos\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)+i\sin\left(\frac{\pi}{2}+\frac{\pi}{2^2}+\frac{\pi}{2^3}+\ldots\right)$$

$$=\cos\left(\frac{\pi/2}{1-1/2}\right)+i\sin\left(\frac{\pi/2}{1-1/2}\right)$$

$$=\cos\pi+i\sin\pi=-1$$

63. ***(a)*** Given, $k\cos x-3\sin x=k+1$

We can write above equation as

$$\frac{k}{\sqrt{k^2+9}}\cos x-\frac{3}{\sqrt{k^2+9}}\sin x=\frac{k+1}{\sqrt{k^2+9}}$$

which possess solution only if

$$-1\le\frac{k+1}{\sqrt{k^2+9}}\le 1$$

$\therefore$ $$\left|\frac{k+1}{\sqrt{k^2+9}}\right|\le 1$$

$\Rightarrow (k+1)^2\le k^2+9$

$\Rightarrow k^2+2k+1\le k^2+9$

$\Rightarrow k\le 4$

$\therefore$ The equation $k\cos x-3\sin x=k+1$ is solvable only if $k\in(-\infty, 4]$.

64. ***(c)*** Let $S_1\equiv x^2+y^2+4x+22y+c=0$

and $S_2\equiv x^2+y^2-2x+8y-d=0$

According to the question, circle S_1 bisects the circumference of the circle.

$\because$ The common chord of the given circle is $S_1-S_2=0$

$\therefore (x^2+y^2+4x+22y+c)-(x^2+y^2-2x+8y-d)=0$

$\Rightarrow 6x+14y+c+d=0 \quad \ldots(i)$

Thus, Eq. (i) passes through centre at the given circle, i.e. $(1, -4)$.

$\therefore 6(1)+14(-4)+c+d=0$

$\Rightarrow 6-56+c+d=0$

$\Rightarrow c+d=50$

65. ***(c)*** Given, $x^2+px+q=0 \quad \ldots(i)$

$\because$ One of above equation is $2+\sqrt{3}$, therefore the other root will be $2-\sqrt{3}$.

$\therefore$ Sum of roots $=(2+\sqrt{3})+(2-\sqrt{3})$

$\Rightarrow 4=-p$ [$\because$ sum of roots of Eq. (i) is $-p$]

$\Rightarrow p=-4$

and product of the roots $=(2+\sqrt{3})(2-\sqrt{3})$

$=(2)^2-(\sqrt{3})^2$

$=4-3=1=q$ [$\because$ product of roots of Eq. (i) is q]

$\therefore q=1$

66. ***(d)*** According to the question, the possible cases arise:

Case I When man invites 3 ladies and woman invites 3 gentlemen, then

Number of ways $={}^4C_3\cdot{}^4C_3=16$

Case II When man invites (2 ladies, 1 gentleman) and woman invites (2 gentlemen, 1 lady), then

Number of ways $=({}^4C_2\cdot{}^3C_1)\cdot({}^4C_2\cdot{}^3C_1)=324$

Case III When man invites (1 lady, 2 gentlemen) and woman invites (2 ladies, 1 gentlemen), then

Number of ways $=({}^4C_1\cdot{}^3C_2)\cdot({}^3C_2\cdot{}^4C_1)=144$

Case IV When man invites 3 men and woman invites 3 ladies, then

Number of ways $={}^3C_3\cdot{}^3C_3=1$

$\therefore$ Total number of ways $=16+324+144+1$

$=485$

67. ***(b)*** According to the question, each term is equal to sum of next two terms.

$\therefore$ $$t_n=t_{n+1}+t_{n+2}$$

$\Rightarrow ar^{n-1}=ar^n+ar^{n+1}$

$\Rightarrow r^2+r-1=0$

$\therefore$ $$r=-\frac{1\pm\sqrt{5}}{2}$$

We can select only positive value of r as GP consist of only positive terms.

$\therefore$ $$r=-\frac{1+\sqrt{5}}{2}=\frac{1}{2}(\sqrt{5}-1)$$

68. ***(b)*** Let $f(x) = 2x^3 + 5$ and $g(x) = 9x^2 - 12x$

$\because$ $f(x)$ increases less rapidly than $g(x)$.

$\therefore \quad f'(x) < g'(x)$

$\Rightarrow \quad 6x^2 < 18x - 12$

$\Rightarrow \quad x^2 - 3x + 2 < 0$

$\Rightarrow \quad (x-1)(x-2) < 0$

$\Rightarrow \quad 1 < x < 2$

$\therefore \quad x \in (1, 2)$

69. ***(a)*** We have, $|x| = 1$

$\therefore \quad x = \pm 1$

Also, we have

$$y = xe^{|x|} = \begin{cases} x \cdot e^{-x}, & -1 \le x < 0 \\ x \cdot e^{x}, & 0 \le x \le 1 \end{cases}$$

$\therefore$ Required area $= \left| \int_{-1}^{0} xe^{-x} dx \right| + \int_{0}^{1} x \cdot e^{x}\, dx$

$= |[-xe^{-x} - e^{-x}]_{-1}^{0}| + [x \cdot e^{x} - e^{x}]_{0}^{1}$

$= |\{(0-1) - (1 \cdot e - e)\}| + \{(e - e) - (0 - 1)\}$

$= 1 + 1 = 2$ sq units

70. ***(b)*** Let $I = \int e^{e^x} e^x\, dx$

Put $e^x = t$

$\Rightarrow \quad e^x\, dx = dt$

$\therefore \quad I = \int e^t\, dt = e^t + C$

$\therefore \quad I = e^{e^x} + C$

71. ***(d)*** We know that, dot and cross can be interchanged without changing the cyclic order in scalar triple product.

$\therefore \mathbf{b} \cdot (\mathbf{a} \times \mathbf{c})$ is different from all the other options.

72. ***(d)*** Since, each vector is perpendicular to the sum of two vectors.

$\therefore \quad \mathbf{p} \cdot (\mathbf{q} + \mathbf{r}) = 0$

$\Rightarrow \quad \mathbf{p} \cdot \mathbf{q} + \mathbf{p} \cdot \mathbf{r} = 0 \quad \ldots(i)$

$\mathbf{q} \cdot (\mathbf{p} + \mathbf{r}) = 0$

$\Rightarrow \quad \mathbf{q} \cdot \mathbf{p} + \mathbf{q} \cdot \mathbf{r} = 0 \quad \ldots(ii)$

and $\mathbf{r} \cdot (\mathbf{p} + \mathbf{q}) = 0$

$\Rightarrow \quad \mathbf{r} \cdot \mathbf{p} + \mathbf{r} \cdot \mathbf{q} = 0 \quad \ldots(iii)$

On adding Eqs. (i), (ii) and (iii), we get

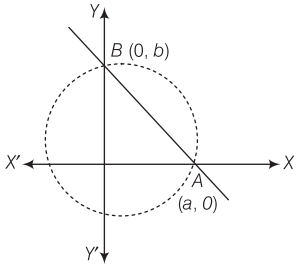
$2(\mathbf{p} \cdot \mathbf{q} + \mathbf{q} \cdot \mathbf{r} + \mathbf{r} \cdot \mathbf{p}) = 0 \quad \ldots(iv)$

Now, $|\mathbf{p} + \mathbf{q} + \mathbf{r}|^2 = |\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2 + 2(\mathbf{p} \cdot \mathbf{q} + \mathbf{q} \cdot \mathbf{r} + \mathbf{r} \cdot \mathbf{p})$

$= |\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2 + 0$ [from Eq. (iv)]

$= |\mathbf{p}|^2 + |\mathbf{q}|^2 + |\mathbf{r}|^2$

73. ***(d)*** Let coordinates of A and B be $(a, 0)$ and $(0, b)$.



$\therefore$ Equation of line AB is $\frac{x}{a} + \frac{y}{b} = 1$.

$\because$ Line passes through the point $(4, 2)$.

$\Rightarrow \quad \frac{4}{a} + \frac{2}{b} = 1 \quad \ldots(i)$

Centre of circle is $\left(\frac{a}{2}, \frac{b}{2}\right)$.

Now, Eq. (i) can be rewritten as $\frac{2}{\left(\frac{a}{2}\right)} + \frac{1}{\left(\frac{b}{2}\right)} = 1$

$\therefore$ Locus of centre of circumference is $\frac{2}{x} + \frac{1}{y} = 1$.

74. ***(b)*** Given, probability that a man can hit a target,

$P(A) = \frac{3}{4}$

$\therefore \quad P(\overline{A}) = 1 - P(A) = 1 - \frac{3}{4} = \frac{1}{4}$

$\because$ He tries 5 times.

$\therefore \quad n = 5$

Let X represent the number of times he hits the targets.

$\therefore \quad P(X = r) = {}^5C_r \left(\frac{3}{4}\right)^r \left(\frac{1}{4}\right)^{5-r}$

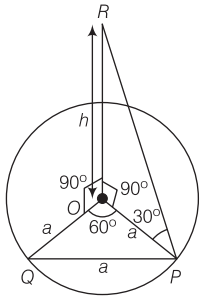
$\therefore \quad P(X = r \le 1) = P(0) + P(1)$

$= \left(\frac{1}{4}\right)^5 + 5\left(\frac{3}{4}\right)^1 \left(\frac{1}{4}\right)^4$

$= \left(\frac{1}{4}\right)^4 \left(\frac{1}{4} + \frac{15}{4}\right) = \left(\frac{1}{4}\right)^4 \times 4$

$= \left(\frac{1}{4}\right)^3$

75. ***(b)*** Let height of the tower be h.



$\because \angle POQ = 60°$

$\therefore \Delta OPQ$ is an equilateral triangle.

$\therefore \quad OP = OQ = PQ = a$

Now, in ΔOPR,

$\tan 30° = \frac{h}{a}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{a}$

$\therefore \quad h = \frac{a}{\sqrt{3}}$

RELATION-FUNCTION
FOR CLASS XII

Topicwise Collection of Best Subjective Problems

1. Let R be the equivalence relation in the set $A = \{0, 1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : 2 \text{ divides } (a - b)\}$. Write the equivalence class $[0]$.

2. If $f : R \to R$ is defined by $f(x) = 3x + 2$, then find $f[f(x)]$.

3. Write fog, if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 8x^3$ and $g(x) = x^{1/3}$.

4. What is the range of the function $f(x) = \dfrac{|x-1|}{x-1}$, $x \neq 1$?

5. If f is an invertible function, defined as $f(x) = \dfrac{3x-4}{5}$, then find $f^{-1}(x)$.

6. If $f : R \to R$ and $g : R \to R$ are given by $f(x) = \sin x$ and $g(x) = 5x^2$, then find $gof(x)$.

7. Let '*' be a binary operation on the set of all non-zero real numbers, given by $a * b = \dfrac{ab}{5}$ for all $a, b \in R - \{0\}$. Find the value of x, if $2 * (x * 5) = 10$.

8. Let '*' be a binary operation on N given by $a * b = \text{LCM}(a, b)$ for all $a, b \in N$. Find $5 * 7$.

9. If the binary operation '*' on the set of integers Z, is defined by $a * b = a + 3b^2$, then find the value of $8 * 3$.

10. If '*' is a binary operation on set Q of rational numbers defined as $a * b = \dfrac{ab}{5}$, then write the identity for '*', if any.

11. If $f : W \to W$ is defined as $f(x) = \begin{cases} x-1, & \text{if } x \text{ is odd} \\ x+1, & \text{if } x \text{ is even} \end{cases}$, then show that f is invertible. Also, find the inverse of f, where W is the set of all whole numbers.

12. If $A = \{1, 2, 3, \ldots, 9\}$ and R is the relation in $A \times A$ defined by $(a, b)\, R\, (c, d)$ and $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$, then prove that R is an equivalence relation. Also, obtain the equivalence class $[(2, 5)]$.

13. Consider $f : R^+ \to [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible with the inverse f^{-1} of f given by $f^{-1}(y) = \sqrt{y-4}$, where R^+ is the set of all non-negative real numbers.

14. Show that $f : N \to N$, given by $f(x) = \begin{cases} x+1, & \text{if } x \text{ is odd} \\ x-1, & \text{if } x \text{ is even} \end{cases}$ is bijective.

15. If $f : R \to R$ is defined as $f(x) = 10x + 7$, then find the function $g : R \to R$, such that $gof = fog = I_R$.

16. Show that the relation S in the set R of real numbers defined as $S = \{(a, b) : a, b \in R \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.

17. If the function $f : R \to R$ is given by $f(x) = \dfrac{x+3}{3}$ and $g : R \to R$ is given by $g(x) = 2x - 3$, then find
(i) fog (ii) gof
Is $f^{-1} = g$?

18. If S is the set of all rational numbers except 1 and '*' is defined on S by $a * b = a + b - ab$, for all $a, b \in S$. Prove that
(i) '*' is a binary operation on S.
(ii) '*' is commutative as well as associative.

19. Consider the binary operation '*' on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min\{a, b\}$. Write operation table of operation '*'.

20. A binary operation '*' on the set $\{0, 1, 2, 3, 4, 5\}$ is defined as $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$.
Show that zero is the identity for this operation and each element a of the set is invertible with $6 - a$, being the inverse of a.

21. If '*' is a binary operation on Q defined by $a * b = \dfrac{3ab}{5}$, then show that '*' is commutative as well as associative. Also, find its identity, if it exists.

22. Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined by $f(x) = \frac{4x+3}{6x-4}$, is one-one and onto. Hence, find the inverse of f.

23. If $f : R \to R$ is the function defined by $f(x) = 4x^3 + 7$, then show that f is a bijective.

24. Show that the relation S in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by $S = \{(a, b) : |a - b|$ is divisible by 4, where $a, b \in Z\}$ is an equivalence relation. Also, find the set of all elements related to A.

25. Show that the function $f : R \to R$ given by $f(x) = ax + b, \forall\ a, b \in R, a \neq 0$, is a bijective.

26. Prove that the relation R on set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b|$ is even$\}$, is an equivalence relation.

27. If $f : N \to N$ is defined by

$$f(x) = \begin{cases} \frac{x+1}{2}, & \text{if } x \text{ is odd} \\ \frac{x}{2}, & \text{if } x \text{ is even} \end{cases} \quad \text{for all } x \in N.$$

Then, find whether the function f is bijective.

28. If N denotes the set of all natural numbers and R is the relation on $N \times N$ defined by $(a, b)\ R\ (c, d)$ when $ad\ \ (b + c) = bc\ (a + d)$, then show that R is an equivalence relation.

29. Let $f : N \to R$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Then show that $f : N \to S$, where S is the range of f, is invertible. Also, find the inverse of f.

30. A binary operation '*' is defined on the set $X = R - \{-1\}$ by $x * y = x + y + xy, \forall\ x, y \in X$. Check whether '*' is commutative and associative. Also, find its identity element and hence find the inverse of each element of X.

Solutions

1. Given, $R = \{(a, b) : 2$ divides $(a - b)\}$
and $A = \{0, 1, 2, 3, 4, 5\}$
Clearly, $[0] = \{b \in A : (0, b) \in R\}$
$= \{b \in A : 2$ divides $(0 - b)\}$
$= \{b \in A : 2$ divides $(-b)\} = \{0, 2, 4\}$
Hence, equivalence class of $[0] = \{0, 2, 4\}$

2. Given, $f(x) = 3x + 2$
$\therefore\ f[f(x)] = f(3x + 2) = 3(3x + 2) + 2$
$= 9x + 6 + 2 = 9x + 8$

3. Given, $f(x) = 8x^3$ and $g(x) = x^{1/3}$
$\therefore\ fog(x) = f[g(x)] = f(x^{1/3}) = 8(x^{1/3})^3 = 8x$

4. Given function is $f(x) = \frac{|x-1|}{x-1}, x \neq 1.$
The above function may be written as

$$f(x) = \begin{cases} \frac{x-1}{x-1}, & \text{if } x > 1 \\ -\frac{(x-1)}{x\ \ 1}, & \text{if } x < 1 \end{cases} \Rightarrow f(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } x < 1 \end{cases}$$

Hence, the range of $f(x)$ is $\{-1, 1\}$.

5. Since, $f(x)$ is an invertible function.
Let $y = \frac{3x-4}{5} \Rightarrow 5y = 3x - 4$
$\Rightarrow\ 3x = 5y + 4 \Rightarrow x = \frac{5y+4}{3}$
$\therefore\ f^{-1}(y) = \frac{5y+4}{3}$
or $f^{-1}(x) = \frac{5x+4}{3}$

6. Given, $f(x) = \sin x$ and $g(x) = 5x^2$
$\therefore\ gof(x) = g[f(x)] = g(\sin x) = 5(\sin x)^2 = 5\sin^2 x$

7. Given, $a * b = \frac{ab}{5}, \forall\ a, b \in R - \{0\}$ ...(i)
and $2 * (x * 5) = 10 \Rightarrow 2 * \left(\frac{x \cdot 5}{5}\right) = 10$
$\Rightarrow\ 2 * x = 10$ [from Eq. (i)]
$\Rightarrow\ \frac{2x}{5} = 10 \Rightarrow x = 25$ [from Eq. (i)]

8. Given, $a * b = \text{LCM}(a, b), \forall\ a, b \in N.$
$\therefore\ 5 * 7 = \text{LCM}(5, 7) = 35$

9. Given, $a * b = a + 3b^2, \forall\ a, b \in Z$
On putting $a = 8$ and $b = 3$, we get
$8 * 3 = 8 + 3 \cdot 3^2 = 8 + 27 = 35$

10. Given binary operation is $a * b = \frac{ab}{5}$.
Let e be the identity element of '*' on Q.
Then, $a * e = a, \forall\ a \in Q$
[by definition of identity element]
$\Rightarrow\ \frac{ae}{5} = a \quad \left[\because a * b = \frac{ab}{5}\right]$
$\Rightarrow\ e = 5$
Hence, 5 is the identity element for '*' on set Q.

11. Given, $f : W \to W$ is defined as

$$f(x) = \begin{cases} x - 1, & \text{if } x \text{ is odd} \\ x + 1, & \text{if } x \text{ is even} \end{cases}$$

One-one function Let $f(x_1) = f(x_2)$
Then, there will be two cases arise.
***Case* I** When x_1 and x_2 are odd, then
$f(x_1) = f(x_2)$
$\Rightarrow\ x_1 - 1 = x_2 - 1$
$\Rightarrow\ x_1 = x_2, \forall\ x_1, x_2 \in W$

***Case* II** When x_1 and x_2 are even, then $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2, \forall\ x_1, x_2 \in W$

So, from case I and case II, we observe that $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2, \forall\ x_1, x_2 \in W$

Hence, $f(x)$ is one-one function.

Onto function Any odd number $2y + 1$ in the co-domain W, is the image of $2y$ in the domain W.

Also, any even number $2y$ in the co-domain W, is the image of $(2y - 1)$ in the domain W.

Thus, every element in W (co-domain) has its image in W(domain).

So, $f(x)$ is onto.

Since, $f(x)$ is one-one and onto.

Therefore, f is bijective, so it is invertible and its inverse exists.

Let $x, y \in W$, such that $f(x) = y$

$\Rightarrow x - 1 = y$, if x is odd and $x + 1 = y$, if x is even

$$\therefore \quad x = \begin{cases} y + 1, & \text{if } y \text{ is odd} \\ y - 1, & \text{if } y \text{ is even} \end{cases}$$

or $$f^{-1}(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

12. Given, a relation R in $A \times A$, where

$A = \{1, 2, 3, ..., 9\}$, defined as

$R = \{((a, b), (c, d)) : a + d = b + c\}$

or $(a, b)\ R\ (c, d)$, if $a + d = b + c$

For R is an equivalence relation, we have to prove that R is reflexive, symmetric and transitive.

Reflexive Let (a, b) be any arbitrary element of $A \times A$.

i.e. $(a, b) \in A \times A$, where $a, b \in A$

Now, $a + b = b + a$ [$\because$ addition is commutative]

$\therefore \quad (a, b)\ R\ (a, b)$

So, R is reflexive.

Symmetric Let $(a, b), (c, d) \in A \times A$, such that $(a, b)\ R\ (c, d)$, then $a + d = b + c$.

$\Rightarrow b + c = a + d \Rightarrow c + b = d + a \Rightarrow (c, d)\ R\ (a, b)$

So, R is symmetric.

Transitive Let $(a, b), (c, d), (e, f) \in A \times A$, such that $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

Then, $a + d = b + c$ and $c + f = d + e$

On adding LHS to LHS and RHS to RHS, we get

$a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e \Rightarrow (a, b)\ R\ (e, f)$

So, R is transitive.

Thus, R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

Now, equivalence class containing an element x of A is given by $\{y : xRy\}$.

Here, $(a, b)\ R\ (c, d)$

Thus, for $(2, 5)$, we will find (c, d) such that $2 + d = 5 + c$

Clearly, $(2, 5)\ R\ (1, 4)$ as $2 + 4 = 5 + 1$

$(2, 5)\ R\ (2, 5)$ as $2 + 5 = 5 + 2$

$(2, 5)\ R\ (3, 6)$ as $2 + 6 = 5 + 3$

$(2, 5)\ R\ (4, 7)$ as $2 + 7 = 5 + 4$

$(2, 5)\ R\ (5, 8)$ as $2 + 8 = 5 + 5$

and $(2, 5)\ R\ (6, 9)$ as $2 + 9 = 5 + 6$

where, $1, 2, 3, ..., 9 \in A$

Hence, equivalence class of $[(2, 5)]$

$= \{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$.

13. Here, function $f : R \to [4, \infty)$ is given by $f(x) = x^2 + 4$.

One-one function Let $x, y \in R^+$, such that $f(x) = f(y)$

$\Rightarrow \quad x^2 + 4 = y^2 + 4 \Rightarrow x^2 = y^2 \Rightarrow x = y$

[$\because$ we take only positive sign as $x, y \in R^+$]

Therefore, f is one-one function.

Onto function For $y \in [4, \infty)$,

Let $y = x^2 + 4$

$\Rightarrow \quad x^2 = y - 4 \geq 0$ [$\because y \geq 4$]

$\Rightarrow \quad x = \sqrt{y - 4} \geq 0$

[$\because$ we take positive sign as $x \in R^+$]

Therefore, for any $y \in R^+$, there exists $x = \sqrt{y - 4} \in R^+$, such that $f(x) = f(\sqrt{y - 4}) = (\sqrt{y - 4})^2 + 4 = y - 4 + 4 = y$.

Therefore, f is onto function.

Thus, f is one-one and onto function. Therefore, f^{-1} exists.

$\therefore \quad x = f^{-1}(y) = \sqrt{y - 4} \Rightarrow f^{-1}(x) = \sqrt{x - 4}$

14. Given function is $f : N \to N$ such that

$$f(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

One-one function From the given function, we observe that there will be two cases arise.

***Case* I** When x is odd, then

Let $f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 + 1 = x_2 + 1 \Rightarrow x_1 = x_2$

$\therefore \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall\ x_1, x_2 \in N$

So, $f(x)$ is one-one function.

***Case* II** When x is even, then

Let $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1 \Rightarrow x_1 = x_2$

$\Rightarrow \quad f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall\ x_1, x_2 \in N$

So, $f(x)$ is one-one function.

Hence, from case I and case II, we observe that

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2, \forall\ x_1, x_2 \in N$

Therefore, $f(x)$ is one-one function.

Onto function To show $f(x)$ is onto, we have to show that its range and codomain are same.

From the definition of given function, we observe that

$f(1) = 1 + 1 = 2, \quad f(2) = 2 - 1 = 1$

$f(3) = 3 + 1 = 4, \quad f(4) = 4 - 1 = 3$ and so on.

So, we get set of natural numbers as the set of values of $f(x)$.

$\Rightarrow \quad$ Range of $f(x) = N$

Also, given that codomain $= N$

[$\because f : N_{\text{domain}} \to N_{\text{codomain}}$]

Thus, range = codomain

Therefore, $f(x)$ is onto function.

Hence, $f(x)$ is bijective.

15. Given, $f(x) = 10x + 7$

Also, $gof = fog = I_R(x)$

Now, $gof = I_r(x)$

$\therefore \quad g[f(x)] = I(x), \forall\ x \in R$

$\Rightarrow \quad g(10x + 7) = x$

$\therefore \quad g(y) = \dfrac{y - 7}{10}$, where $y = 10x + 7$

or $$g(x) = \frac{x - 7}{10}$$

16. Given relation is $S = \{(a, b) : a, b \in R \text{ and } a \le b^3\}$.

Reflexive As $\frac{1}{3} \le \left(\frac{1}{3}\right)^3$, where $\frac{1}{3} \in R$, is not true.

$\Rightarrow \left(\frac{1}{3}, \frac{1}{3}\right) \notin S$

Therefore, S is not reflexive.

Symmetric As $-2 \le (3)^3$, where $-2, 3 \in R$, is true but $3 \le (-2)^3$ is not true, i.e. $(-2, 3) \in S$ but $(3, -2) \notin S$

Therefore, S is not symmetric.

Transitive As $3 \le \left(\frac{3}{2}\right)^3$ and $\frac{3}{2} \le \left(\frac{4}{3}\right)^3$, where $3, \frac{3}{2}, \frac{4}{3} \in R$, are true but $3 \le \left(\frac{4}{3}\right)^3$ is not true.

i.e. $\left(3, \frac{3}{2}\right) \in S$ and $\left(\frac{3}{2}, \frac{4}{3}\right) \in S$ but $\left(3, \frac{4}{3}\right) \notin S$

Therefore, S is not transitive.

Hence, S is neither reflexive nor symmetric nor transitive.

17. Given, $f : R \to R$ such that $f(x) = \frac{x + 3}{3}$

and $g : R \to R$ such that $g(x) = 2x - 3$.

(i) $fog\,(x) = f[g(x)] = f(2x - 3) = \frac{(2x - 3) + 3}{3}$

$\Rightarrow \quad fog(x) = \frac{2x}{3}$

(ii) $gof\,(x) = g[f(x)] = g\left(\frac{x + 3}{3}\right) = \left[2\left(\frac{x + 3}{3}\right)\right] - 3$

$= \frac{2x + 6}{3} - 3 = \frac{2x + 6 - 9}{3}$

$\Rightarrow \quad gof\,(x) = \frac{2x - 3}{3}$

Now, we find f^{-1}.

For that, let $y = \frac{x + 3}{3}$

$\Rightarrow \quad 3y = x + 3 \Rightarrow x = 3y - 3$

$\therefore \quad f^{-1}(y) = 3y - 3 \quad [\because x = f^{-1}(y)]$

or $\quad f^{-1}(x) = 3x - 3$

But $\quad g(x) = 2x - 3$

$\therefore \quad f^{-1} \ne g$

18. Given, '*' is defined as $a * b = a + b - ab, \forall\ a, b \in S$.

(i) We know that, addition of two rational numbers is a rational number. Also, multiplication of two rational numbers is a rational number.

Here, a and b are rational numbers other than 1. So, $a + b$ and ab are also rational numbers.

$\therefore\ a + b - ab$ is also a rational number.

[$\because$ difference of two rational numbers is also a rational number]

Hence, '*' is a binary operation on set S.

(ii) **Commutative** $a * b = a + b - ab = b + a - ba$

[$\because$ rational number is commutative with respect to addition and multiplication]

$\Rightarrow \quad a * b = b * a$

Hence, '*' is commutative.

Associative $(a * b) * c = (a + b - ab) * c$

$= a + b - ab + c - (a + b - ab)\,c$

$= a + b + c - ab - bc - ac + abc \quad \ldots(i)$

and $a * (b * c) = a * (b + c - bc)$

$= a + b + c - bc - a\,(b + c - bc)$

$= a + b + c - ab - bc - ac + abc \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$(a * b) * c = a * (b * c)$

Hence, '*' is associative.

19. Given binary operation is $a * b = \min\{a, b\}$ defined on the set $\{1, 2, 3, 4, 5\}$.

The operation table for operation '*' is as follows :

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

[$\because 1 * 1 = \min\{1, 1\} = 1; 1 * 2 = \min\{1, 2\} = 1, \ldots,$
$5 * 4 = \min\{5, 4\} = 4; 5 * 5 = \min\{5, 5\} = 5$]

20. Given, $a * b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \ge 6 \end{cases}$

The operation table for '*' is as follows :

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

[$\because 0 + 0 < 6 \Rightarrow 0 + 0 = 0; 0 + 1 < 6 \Rightarrow 0 + 1 = 1; \ldots;$
$1 + 4 < 6 \Rightarrow 1 + 4 = 5; 1 + 5 \ge 6 \Rightarrow 1 + 5 - 6 = 0; \ldots$]

From the table, we note that $a * 0 = 0 * a = a$

Hence, 0 is the identity for an operation.

Also, $1 * 5 = 0, 2 * 4 = 0, 3 * 3 = 0, 4 * 2 = 0, 5 * 1 = 0$

Hence, inverse of 1 is 5, i.e. for element a, $6 - a$ is its inverse.

21. Given binary operation is $a * b = \frac{3ab}{5}; a, b \in Q$.

Commutative $a * b = \frac{3ab}{5}$ [given] and $b * a = \frac{3ba}{5}$

[by using definition of '*']

$\because \frac{3ab}{5} = \frac{3ba}{5}, \forall\ a, b \in Q$ is true.

$\therefore \quad a * b = b * a, \forall\ a, b \in Q$

So, '*' is commutative.

Associative $a * (b * c) = a * \left(\frac{3bc}{5}\right) \quad \left[\because a * b = \frac{3ab}{5}\right]$

$= \frac{3a\left(\frac{3bc}{5}\right)}{5} = \frac{9abc}{25}$

and $(a * b) * c = \left(\frac{3ab}{5}\right) * c \quad \left[\because a * b = \frac{3ab}{5}\right]$

$= \frac{3\left(\frac{3ab}{5}\right)c}{5} = \frac{9abc}{25}$

Clearly, $a * (b * c) = (a * b) * c, \forall\ a, b, c \in Q$.

So, '*' is associative.

Existence of identity Let e be the identity element of '*' on Q. Then, by definition of identity element, we must have $a * e = e * a = a, \forall\ a \in Q$

Let $a * e = a, \forall\ a \in Q$

$\Rightarrow \frac{3ae}{5} = a \quad \left[\because a * b = \frac{3ab}{5}\right]$

$\Rightarrow e = \frac{5}{3} \in Q$

$\therefore e = \frac{5}{3}$ is the identity element '*' defined on Q.

22. Given, $f(x) = \frac{4x + 3}{6x - 4}$

Let $x_1, x_2 \in A = R - \left\{\frac{2}{3}\right\}; x_1 \neq x_2$

One-one function Consider $f(x_1) = f(x_2)$

$\therefore \frac{4x_1 + 3}{6x_1 - 4} = \frac{4x_2 + 3}{6x_2 - 4}$

$\Rightarrow (4x_1 + 3)(6x_2 - 4) = (4x_2 + 3)(6x_1 - 4)$

$\Rightarrow 24x_1x_2 - 16x_1 + 18x_2 - 12 = 24x_1x_2 - 16x_2 + 18x_1 - 12$

$\Rightarrow -34x_1 = -34x_2 \Rightarrow x_1 = x_2$

So, f is one-one function.

Onto function Let $y = \frac{4x + 3}{6x - 4}$

$\Rightarrow 6xy - 4y = 4x + 3$

$\Rightarrow (6y - 4)x = 3 + 4y$

$\Rightarrow x = \frac{3 + 4y}{6y - 4}$ and $y \neq \frac{4}{6}$, i.e. $y \neq \frac{2}{3}$

$\therefore y \in R - \left\{\frac{2}{3}\right\}$

Thus, f is onto function.

Since, f is one-one and onto function.

Thus, f is bijective function, so its inverse exists.

$\therefore x = f^{-1}(y) = \frac{3 + 4y}{6y - 4} \Rightarrow f^{-1}(x) = \frac{3 + 4x}{6x - 4}$

23. Given function is $f : R \to R$ such that $f(x) = 4x^3 + 7$.

For f is bijective, we have to show that f is one-one and onto function.

One-one function Let $f(x_1) = f(x_2), \forall\ x_1, x_2 \in R$

$\Rightarrow 4x_1^3 + 7 = 4x_2^3 + 7$

$\Rightarrow 4x_1^3 = 4x_2^3 \Rightarrow x_1^3 - x_2^3 = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$\Rightarrow$ Either $x_1 - x_2 = 0$ ...(i)

or $x_1^2 + x_1x_2 + x_2^2 = 0$ ...(ii)

But Eq. (ii) is not possible as $x_1, x_2 \in R$.

$\therefore x_1 - x_2 = 0 \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2, \forall\ x_1, x_2 \in R$

Therefore, $f(x)$ is one-one function.

Onto function Given, codomain of $f(x) = R$.

Now, let $y = 4x^3 + 7$

$\Rightarrow 4x^3 = y - 7 \Rightarrow x^3 = \frac{y - 7}{4}$

$\Rightarrow x = \left(\frac{y - 7}{4}\right)^{1/3}$ [taking cube root] ...(iii)

From Eq. (iii), it is clear that for every $y \in R$, we get $x \in R$.

$\therefore$ Range of $f(x) = R$

Thus, range of $f(x)$ = co-domain of $f(x)$

$\Rightarrow f(x)$ is onto function.

Since, $f(x)$ is both one-one and onto, so it is a bijective.

24. Given relation is

$S = \{(a, b) : |a - b|$ is divisible by 4, where $a, b \in Z\}$

and $A = \{x : x \in Z$ and $0 \leq x \leq 12\}$

Now, A can be written as

$A = \{0, 1, 2, 3, ..., 12\}$

For R to be an equivalence relation, we have to prove that R is reflexive, symmetric and transitive.

Reflexive As for any $x \in A$, we get $|x - x| = 0$, which is divisible by 4.

$\Rightarrow (x, x) \in S, \forall\ x \in A$

Therefore, S is reflexive.

Symmetric As for any $(x, y) \in S$, we get $|x - y|$ is divisible by 4. [by using definition of given relation]

$\Rightarrow |x - y| = 4\lambda$, for some $\lambda \in Z$

$\Rightarrow |y - x| = 4\lambda$, for some $\lambda \in Z$

$\Rightarrow (y, x) \in S$

Thus, $(x, y) \in S \Rightarrow (y, x) \in S, \forall\ x, y \in Z$

Therefore, S is symmetric.

Transitive For any $(x, y) \in S$ and $(y, z) \in S$, we get $|x - y|$ is divisible by 4 and $|y - z|$ is divisible by 4. [by using definition of given relation]

$\Rightarrow |x - y| = 4\lambda$ and $|y - z| = 4\mu$, for some $\lambda, \mu \in Z$

Now, $x - z = (x - y) + (y - z) = \pm 4\lambda \pm 4\mu = \pm 4(\lambda + \mu)$

$\Rightarrow |x - z|$ is divisible by 4.

$\Rightarrow (x, z) \in S, \forall\ x, z \in A$

Thus, $(x, y) \in S$ and $(y, z) \in S$

$\Rightarrow (x, z) \in S, \forall\ x, y, z \in Z$

Therefore, S is transitive.

Since, S is reflexive, symmetric and transitive.

So, it is an equivalence relation.

Now, set of all elements related to A

$= \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

25. Given function $f : R \to R$ is such that

$f(x) = ax + b; \forall\ a, b \in R, a \neq 0.$

For $f(x)$ is a bijective, we have to prove that $f(x)$ is both one-one and onto function.

One-one function Let $f(x_1) = f(x_2), \forall\ x_1, x_2 \in R$

$\Rightarrow ax_1 + b = ax_2 + b$

$\Rightarrow ax_1 = ax_2 \Rightarrow x_1 = x_2$

Thus, $f(x_1) = f(x_2), \forall\ x_1, x_2 \in R \Rightarrow x_1 = x_2$

Therefore, $f(x)$ is one-one function.

Onto function Let $y = ax + b \Rightarrow x = \frac{y - b}{a}$ ...(i)

From Eq. (i), it is observed that for every $y \in R, x \in R$

$\therefore$ Range of $f(x) = R$

Also, given that codomain $= R$

Thus, range of $f(x)$ = codomain of $f(x)$

Therefore, $f(x)$ is onto function.

As, $f(x)$ is both one-one and onto, so it is a bijective function.

26. Given relation is $R = \{(a, b) : |a - b| \text{ is even}\}$ defined on set $A = \{1, 2, 3, 4, 5\}$.

For R is an equivalence relation, we have to prove that R is reflexive, symmetric and transitive.

Reflexive As $|x - x| = 0$ is even, $\forall\ x \in A$.

$\Rightarrow \quad (x, x) \in R, \forall\ x \in A$

Therefore, R is reflexive.

Symmetric As $(x, y) \in R$

$\Rightarrow$ $|x - y|$ is even. [by definition of given relation]

$\Rightarrow$ $|y - x|$ is also even. [$\because |a| = |-a|, \forall\ a \in R$]

$\Rightarrow (y, x) \in R, \forall\ x, y \in A$

Thus, $(x, y) \in R \Rightarrow (y, x) \in R, \forall\ x, y \in A$

Therefore, R is symmetric.

Transitive As $(x, y) \in R$ and $(y - z) \in R$

$\Rightarrow |x - y|$ is even and $|y - z|$ is even.

[by using definition of given relation]

Now, $|x - y|$ is even.

$\Rightarrow$ x and y both are even or odd.

Also, $|y - x|$ is even.

$\Rightarrow y$ and x both are even or odd.

Then, two cases arise :

***Case* I** When y is even, then

$(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow |x - y|$ is even and $|y - z|$ is even.

$\Rightarrow x$ is even and z even.

$\Rightarrow |x - z|$ is even.

[$\because$ difference of two even numbers is also even]

$\Rightarrow \quad (x, z) \in R$

***Case* II** When y is odd, then

$(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow |x - y|$ is even and $|y - z|$ is even.

$\Rightarrow x$ is odd and z is odd.

$\Rightarrow |x - z|$ is even.

[$\because$ difference of two odd numbers is also even]

$\Rightarrow (x, z) \in R$

Thus, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R, \forall\ x, y, z \in A$

Therefore, R is transitive.

Since, R is reflexive, symmetric and transitive, so it is an equivalence relation.

27. Given function is $f : N \to N$ such that

$$f(x) = \begin{cases} \dfrac{x + 1}{2}, & \text{if } x \text{ is odd} \\ \dfrac{x}{2}, & \text{if } x \text{ is even} \end{cases}$$

One-one function Now, $f(1) = \dfrac{1 + 1}{2} = \dfrac{2}{2} = 1$

$\left[\text{put } x = 1 \text{ in } f(x) = \dfrac{x + 1}{2}\right]$

and $\quad f(2) = \dfrac{2}{2} = 1 \quad \left[\text{put } x = 2 \text{ in } f(x) = \dfrac{x}{2}\right]$

So, $f(x)$ is not one-one function because for two distinct values of domain (N), $f(x)$ has same image.

Onto function If x is an odd natural number, then $2x - 1$ is also an odd natural number.

Now, $\quad f(2x - 1) = \dfrac{2x - 1 + 1}{2} = x \quad \ldots(i)$

It means that for every odd number, we get an image of even natural number.

Again, if x is even natural number, then $2x$ is also even natural number.

Then, $\quad f(2x) = \dfrac{2x}{2} = x \quad \ldots(ii)$

It means that for every even number, we get an image of odd number.

From Eqs. (i) and (ii), we observe that for each x (whether even or odd), there exists its pre-image in N.

$\therefore$ Range = Co-domain

Therefore, f is onto function.

Since, $f(x)$ is not one-one but it is onto.

So, it is not a bijective function.

28. We have, a relation R on $N \times N$ defined by $(a, b)\ R\ (c, d)$, if $ad\ (b + c) = bc\ (a + d)$.

For R is an equivalence relation, we have to prove that R is reflexive, symmetric and transitive.

Reflexive Let $(a, b) \in N \times N$ be any arbitrary element.

We have to show that $(a, b)\ R\ (a, b)$, i.e. to show $ab\ (b + a) = ba\ (a + b)$ which is trivally true as natural numbers are commutative under usual multiplication and addition.

Since, $(a, b) \in N \times N$ was arbitrary. Therefore, R is reflexive.

Symmetric Let $(a, b), (c, d) \in N \times N$ such that $(a, b)\ R\ (c, d)$.

i.e. $\quad ad\ (b + c) = bc\ (a + d) \quad \ldots(i)$

To show, $(c, d)\ R\ (a, b)$, i.e. to show $cb\ (d + a) = da\ (c + b)$

From Eq. (i), we get

$ad\ (b + c) = bc\ (a + d) \Rightarrow da\ (c + b) = cb\ (d + a)$

[$\because$ natural numbers are commutative under usual addition and multiplication]

Transitive Let $(a, b), (c, d), (e, f) \in N \times N$ such that $(a, b)\ R\ (c, d)$ and $(c, d)\ R\ (e, f)$.

$\because\ (a, b)\ R\ (c, d) \Rightarrow ad\ (b + c) = bc\ (a + d)$

$\Rightarrow \quad \dfrac{b + c}{bc} = \dfrac{a + d}{ad} \Rightarrow \dfrac{1}{b} + \dfrac{1}{c} = \dfrac{1}{a} + \dfrac{1}{d} \quad \ldots(ii)$

and $(c, d)\ R\ (e, f) \Rightarrow cf\ (d + e) = de\ (c + f)$

$\Rightarrow \quad \dfrac{d + e}{de} = \dfrac{c + f}{cf} \Rightarrow \dfrac{1}{d} + \dfrac{1}{e} = \dfrac{1}{c} + \dfrac{1}{f} \quad \ldots(iii)$

On adding Eqs. (ii) and (iii), we get

$$\left(\frac{1}{b} + \frac{1}{c}\right) + \left(\frac{1}{d} + \frac{1}{e}\right) = \left(\frac{1}{a} + \frac{1}{d}\right) + \left(\frac{1}{c} + \frac{1}{f}\right)$$

$\Rightarrow \quad \dfrac{1}{b} + \dfrac{1}{e} = \dfrac{1}{a} + \dfrac{1}{f} \Rightarrow \dfrac{e + b}{be} = \dfrac{f + a}{af}$

$\Rightarrow \quad af\ (e + b) = be\ (f + a)$

$\Rightarrow \quad af\ (b + e) = be\ (a + f) \Rightarrow (a, b)\ R\ (e, f)$

$\Rightarrow R$ is transitive.

Thus, R is reflexive, symmetric and transitive.

Hence, R is an equivalence relation.

29. We have a mapping $f : N \to R$ given by

$$f(x) = 4x^2 + 12x + 15$$

For f is invertible, we have to show that f is one-one and onto function.

One-one function Let $x_1, x_2 \in N$, then

Consider $f(x_1) = f(x_2)$

$\therefore \quad 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow \quad 4x_1^2 + 12x_1 = 4x_2^2 + 12x_2$

$\Rightarrow \quad 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0$

$\Rightarrow \quad 1(x_1 - x_2)(x_1 + x_2) + 3(x_1 - x_2) = 0$ [divide by 4]

$\Rightarrow \quad (x_1 - x_2)(x_1 + x_2 + 3) = 0$

$\Rightarrow$ Either $(x_1 - x_2) = 0$

or $x_1 + x_2 + 3 = 0$

Since, $x_1, x_2 \in N$, so $x_1 + x_2 + 3 = 0$ cannot possible.

$\therefore \quad x_1 = x_2$

So, f is one-one function.

Onto function Obviously, $f : N \to S$ is an onto function because codomain S is the range of f.

$\therefore f : N \to S$ is one-one and onto function.

$\Rightarrow f$ is invertible function, so its inverse exists.

Let $f(x) = y$, then

$$y = 4x^2 + 12x + 15$$

$$\Rightarrow \quad y = (2x)^2 + 2 \cdot 2x \cdot 3 + 3^2 + 6$$

$$\Rightarrow \quad y = (2x + 3)^2 + 6$$

$$\Rightarrow \quad (2x + 3)^2 = y - 6$$

$$\Rightarrow \quad 2x + 3 = \sqrt{y - 6} \Rightarrow x = \frac{\sqrt{y - 6} - 3}{2}$$

$$\therefore \quad f^{-1}(x) = \frac{\sqrt{x - 6} - 3}{2}$$

30. Given, '*' is a binary operation defined on the set $X = R - \{-1\}$ such that

$$x * y = x + y + xy, \forall\ x, y \in X$$

Commutative Let $x_1, x_2 \in X$ be any arbitrary elements.

Then, $x_1 * x_2 = x_1 + x_2 + x_1 x_2$ [by using definition of '*']

$= x_2 + x_1 + x_2 x_1$

[$\because$ addition and multiplication are commutative on the set of real numbers]

$= x_2 * x_1$

$\because x_1, x_2 \in X$ are arbitrary.

$\therefore$ '*' is commutative.

Associative Let $x_1, x_2, x_3 \in X$ be any arbitrary elements.

Then, $x_1 * (x_2 * x_3) = x_1 * (x_2 + x_3 + x_2 x_3)$

[by using definition of '*']

$= x_1 + x_2 + x_3 + x_2 x_3 + x_1 (x_2 + x_3 + x_2 x_3)$

$= x_1 + x_2 + x_3 + x_2 x_3 + x_1 x_2 + x_1 x_3 + x_1 x_2 x_3$ ...(i)

[$\because$ multiplication is distributive over addition on the set of real numbers]

and $(x_1 * x_2) * x_3 = (x_1 + x_2 + x_1 x_2) * x_3$

[by using definition of '*']

$= x_1 + x_2 + x_1 x_2 + x_3 + (x_1 + x_2 + x_1 x_2)\, x_3$

$= x_1 + x_2 + x_1 x_2 + x_3 + x_1 x_3 + x_2 x_3 + x_1 x_2 x_3$

[$\because$ multiplication is distributive over addition on the set of real numbers]

$= x_1 + x_2 + x_3 + x_2 x_3 + x_1 x_2 + x_1 x_3 + x_1 x_2 x_3$ ...(ii)

[$\because$ addition and multiplication is commutative on the set of real numbers]

From Eqs. (i) and (ii), we get

$$x_1 * (x_2 * x_3) = (x_1 * x_2) * x_3$$

$\because x_1, x_2$ and x_3 are arbitrary.

$\therefore$ '*' is associative.

Existence of identity Let $e \in X$ be an identity element of X.

Then, $\quad x * e = x = e * x, \forall\ x \in X$

Now, $\quad x * e = x \Rightarrow x + e + xe = c$

[by using definition of '*']

$\Rightarrow \quad e + ex = 0 \Rightarrow e(1 + x) = 0$

$\Rightarrow \quad e = 0$ [$\because x \neq -1$]

Similarly, $e * x = x \Rightarrow e = 0$

Thus, $e = 0 \in X$ is the identity element for X.

Existence of inverse Let $x \in X$ be any arbitrary element.

Now, if possible, let $y \in X$ be the inverse of x.

Then, $\quad x * y = 0 = y * x$

Now, $\quad x * y = 0$

$\Rightarrow \quad x + y + xy = 0$ [by using definition of '*']

$\Rightarrow \quad y(1 + x) = -x \Rightarrow y = \frac{-x}{1 + x} \in X, x \neq -1$

Thus, for all $x \in X$, there exists

$y = \frac{-x}{1 + x} \in X, x \neq -1$ such that $x * y = 0 = y * x$.

Hence, inverse of $x \in X$ is $\frac{-x}{1 + x}, x \neq -1$.

Mind Twister

1. The egg vendor calls on his first customer and sells half his eggs and half an egg. To the second customer, he sells half of what he had left and half an egg, and to the third customer, he sells half of what he had then left and half an egg. By the way, he did not break any eggs. In the end, three eggs were remaining.

   How many did he start out with?

2. Vikram Chadda has two children and atleast one of them is a girl. What is the probability that both children are girls?

3. A standard deck of cards has 52 cards. What is the probability that six cards drawn at random will all be black cards?

4. If you were to count one number per second and counted seven hours per day, how long would it take you to count to a Billion?

5. Is 1000009 a prime number?

6. $17 \cdot x = 17 + x$

   What is the value of x?

SUPER SUBSCRIPTION OFFER

NOW GET YOUR SPECTRUM AT YOUR DOOR STEP...

Magazine	Year	No. of Copies	Amount	Company Discount (Less)	Courier Charges (Add)	Final Amount
P/C/M (anyone)	1	12	390/-	137/-	360/-	613/-
P/C/M (anyone)	2	24	780/-	312/-	720/-	1188/-
P/C/M (all three)	1	36	1170/-	410/-	1080/-	1840/-
P/C/M (all three)	2	72	2340/-	936/-	2160/-	3564/-

ALL PARTICULARS EXCEPT EMAIL TO BE FILLED IN BLOCK LETTERS

Name ..... Father's Name .....

DOB ..... Name of the Institute *(in which you are preparing for exam)* .....

Name of the School ..... Address .....

.....

Phone ..... E-mail .....

I am enclosing DD No. ..... Dated..... branch (Specify Bank)

..... favouring **Arihant Media Promoters** to be drawn at MEERUT

for Rs .....

Please complete the order form and send to :

Arihant Media Promoters, Kalindi, T P Nagar, Meerut-2

TERMS AND CONDITIONS : Please allow 4–5 weeks for processing of your subscription; No cancellation will be entertained after the acceptance of subscription; Please mention your name and address on the reverse of your DD; All disputes subject to Meerut (UP) jurisdiction only

You can also subscribe Magazines online at www.arihantbooks.com

For more information, contact : **+91-9219641347**

FUN ARCADE

CR RAO

CR Rao was born on 10 September 1920 in Huvanna Hadagali, now in Karnataka state. He studied in schools at Gudur, Nuzvid, Nandigama and Visakhapatnam, in Andhra Pradesh and completed his MA degree in Mathematics at the Andhra university in Waltair, Andhra Pradesh. He also did his MA degree in Statistics from Calcutta university in Kolkata, West Bengal.

CR Rao worked at the Indian Statistical Institute, India for 40 years before he took retirement. He accepted University professorship at the university of Pittsburgh after his retirement. He worked for eight years at the university of Pittsburgh and moved to the Pennsylvania state university as Eberly professor of Statistics, where he continues to work as the Director of the center for multi-variate analysis. Till now, he received 31 Honorary Doctoral degrees from universities in 18 countries spanning six continents.

C. Radhakrishna Rao is among the world leaders in statistical science over the last six decades. His research, scholarship and professional services have had a profound influence on theory and applications of statistics.

Technical terms such as, Cramer-Rao inequality, Rao - Blackwellisation, Rao's score test, Fisher-Rao and Rao theorems on second order efficiency of an estimator, Rao metric and distance, analysis of dispersion (MANOVA) and canonical variate analysis and G-inverse of matrices appear in all standard books on Statistics.

He is the author of 14 books and about 350 research papers. Three of his books have been translated into several European, Chinese and Japanese languages.

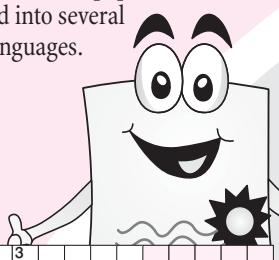
CROSSWORD

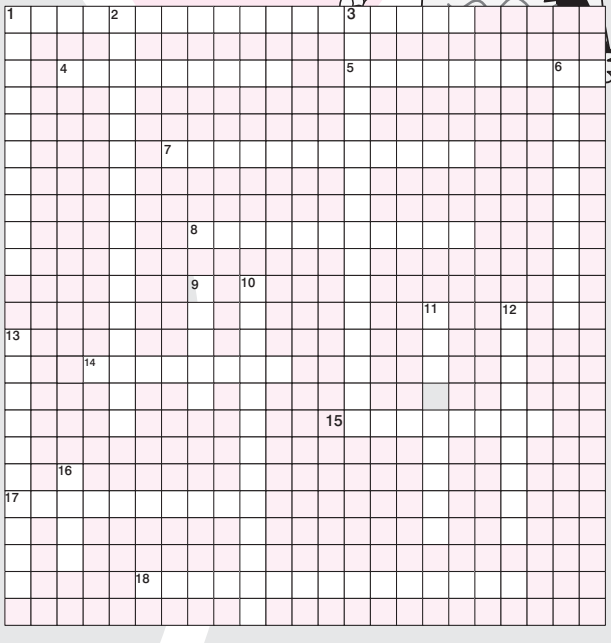
ACROSS

1. In a cylinder, if the fixed curve is a hyperbola and the fixed line to which the generator are parallel, is perpendicular to the plane of the hyperbola. Then, this cylinder is ..... .
4. A line generates a surface is ..... .
5. The path traced out by a projectile.
7. A pyramid which has its vertex directly above the centre of its base.
8. A simple closed curve, i.e. curve which has no ends and does not intersect itself.
14. A constrained optimisation problem satisfied simultaneously is said to be ..... .
15. A line segment that is one-half of an axis of a conic.
17. Another name of countable.
18. A solution of linear differential equation containing the same number of arbitrary constants as the order of the equation.

DOWN

1. A theory or formula that has to be prove but is assumed to be true for the purpose of the argument.
2. The experiments which when repeated under identical conditions, don't produce the same outcomes everytime but the outcome in a trial is one of the several possible outcomes.
3. The branch of calculus which can be used for finding areas, volumes etc.
6. A prism in which its lateral face are perpendicular to its bases.
9. A curve on the surface of a right circular cylinder that cuts the generators of the cylinder at a constant angle.
10. A frequency distribution in which each upper limit of each class is excluded and lower limit is included.
11. When the independent variables is not the only term on one side of an equation.
12. A circle whose radius is imaginary.
13. Who is known as father of the integral calculus.
16. The absolute value of the difference between the left hand and right hand side of the given function.





Puzzles to Puzzle You

1. We have a circular dining table made of marble which had come down to us as a family heirloom. We also have some beautiful bone-china saucers that I recently brought from Japan.
Diameter of our table top is fifteen times the diameter of our saucers which are also circular. We would like to place the saucers on the table so that they neither overlap each other nor the edge of the table.
How many can we place in this manner?

2. A friend of mine in Bangalore owns a horse-driven carriage. It was found that the fore wheels of the carriage make four more revolutions than the hind wheel in going 96 feet. However, it was also found that if the circumference of the fore wheel was 3/2 as great and of the hind wheel 4/3 as great, then the fore wheel would make only 2 revolutions more than hind wheel in going the same distance of 96 feet. Can you find the circumference of each wheel?

3. A number of cats got together and decided to kill between them 999919 mice. Every cat killed an equal number of mice.
How many cats do you think there were? Oh, by the way, let me clarify just two points, it is not one cat killed the lot, because I have said 'cats' and it is not 999919 cats each killed one mouse, because I have used the word 'mice'. I can give you just one clue, each cat killed more mice than there were cats.

4. We all know that a chess board has 64 squares. This can be completely covered by 32 cardboard rectangles, each cardboard covering just 2 squares.
Supposing we remove two squares of the chess board at diagonally opposite corners, can we cover the modified board with 31 rectangles? If it can be done, how can we do it? And if it cannot be done, prove it impossible.

5. When my uncle in Madura died recently, he left a will, instructing his executors to divide his estate of ₹1920000. In this manner, every son should receive three times as much as a daughter and that every daughter should get twice as much as their mother.
What is my aunt's share?

6. It is a small town railway station and there are 25 stations on that line. At each of the 25 stations, the passengers can get tickets for any of the other 24 stations.
How many different kinds of tickets do you think the booking clerk has to keep?

SUDOKU

		1			4			
3				5		9		2
			9				7	3
					3	8		
7	9			1				
5	2				8			
				4				5
			1	6				4

March 2017 issue Answer

Crossword

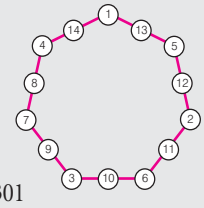
								1 T						2 V					
								R					3 Z	E	R	O			
					4 V	A	R	I	A	N	C	E		R					
								V						T					
	5 A	R	R	A	Y			6 I	S	O	S	7 C	E	L	E	S			
	D							A				O			X				
	J				8 N	U	L	L		9 E		N				10 N			11 E
	O									Q		C				O			X
12 B	I	J	E	13 C	T	14 I	V	E		U		E				T			H
	N			O		M				I		N				D			A
	T			N		A		15 S		16 L	A	T	U	17 S	R	E	C	T	U	M
		18 I		J		G		K		A		R		E		F			S
		N		U		I		E		T		I		C		I			T
		R		G		N		W		E		C		A		N			I
		A		A		A		L		R				N		E			V
		19 D	E	T	E	R	M	I	N	A	N	T		T		D			E
		I		E		Y		N		L									
		U						E											
20 A	B	S	C	I	S	S	A	S											

Puzzles to Puzzle you Answers March

1. 55 km/h

2. 5/7

3.



4. 301

5. 5, 3, 1, 4, 2

6. 7 : 15 am, 8 : 15 am

7. 7, 10 and 16

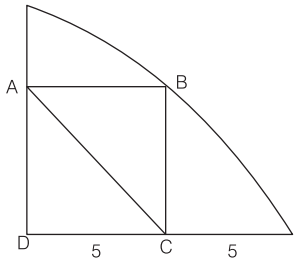
SUDOKU

6	4	2	1	8	9	5	7	3
5	3	9	7	6	2	1	8	4
1	8	7	4	5	3	9	6	2
7	2	5	3	4	1	6	9	8
8	1	4	9	7	6	2	3	5
3	9	6	8	2	5	7	4	1
2	7	3	6	1	8	4	5	9
4	5	8	2	9	7	3	1	6
9	6	1	5	3	4	8	2	7

HURRY! WIN ₹1000 IN CASH

JUST SOLVE & SEND
KNOWLEDGE COEFFICIENT
QUIZZER (NO. 29)

1. A rectangle is inscribed in the quadrant of a circle as shown.



If the unit distances are indicated, can you accurately determine the length of the diagonal AC?

2. Find the unit digit of $(3127)^{173}$.

3. What will be the value of P, if $4864 \times 9P2$ is divisible by 12?

4. A number when divided by 3 leaves a remainder 1. When the quotient is divided by 2, it leaves a remainder 1. What will be the remainder when the number is divided by 6?

5. With the money I have, I can buy 50 pens or 150 pencils. I kept 10% aside for taxi fare. With the remaining, I purchased 54 pencils and P pens. What is the value of P?

6. Find the wrong term in the letter-number series given below.

G4T, J10R, M20P, P43N, S90L

7. If $4©5 = 189$ and $10©8 = 1512$, then what will be $6©9$?

8. A year starting with Monday and ending with Tuesday. How many days are there from 16th January to 15th March of that year.

9. In a 1000 m race, Usha beats Shiny by 50 m. In the same race, by what time margin Shiny beat Mercy who runs at 4 m/s?

10. In the given series, find the missing term.

206, 219, 240, 269, ?

KNOWLEDGE Coefficient Quizzer (No. 29)

Write your answers on separate sheets and send to us along with this coupon

(ALL PARTICULARS TO BE FILLED UP IN BLOCK LETTERS)

Name ..... Date of Birth .....
Class ..... Name of the School ..... City .....
Address of Communication.....
.....
..... Telephone .....
E-mail ..... Hobbies .....

Winner of **Knowledge Coefficient** Quizzer (No. 28) (March Issue)
Lokesh Gola (Aligarh)

Cut, fill and send to :
Editor **Mathematics Spectrum**
Arihant Media Promoters
c/o Arihant Prakashan
Kalindi, TP Nagar, Meerut (UP) - 250002

SOLVED PAPERS & PRACTICE SETS FOR

JEE MAIN & ADVANCED

38 Years' Solved Papers (JEE Main & Advanced)

38 YEARS' SOLVED PAPERS IIT JEE PHYSICS
38 YEARS' SOLVED PAPERS IIT JEE CHEMISTRY
38 YEARS' SOLVED PAPERS IIT JEE MATHEMATICS
38 YEARS' अध्यायवार सॉल्वड पेपर्स IIT JEE भौतिकी
38 YEARS' अध्यायवार सॉल्वड पेपर्स IIT JEE रसायन
38 YEARS' अध्यायवार सॉल्वड पेपर्स IIT JEE गणित

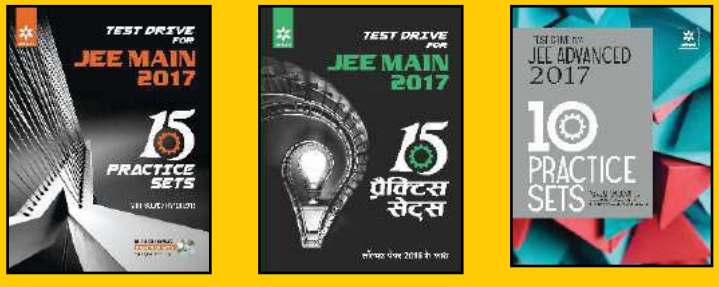
14 YEARS' SOLVED PAPERS

14 YEARS' SOLVED PAPERS 2003-2016 IIT JEE (JEE Main & Advanced)
14 YEARS' UNSOLVED QUESTION PAPERS 2003-2016 IIT JEE (JEE Main & Advanced)
14 YEARS' OBJECTIVE SOLVED PAPERS 2003-2016 IIT JEE (JEE Main & Advanced)

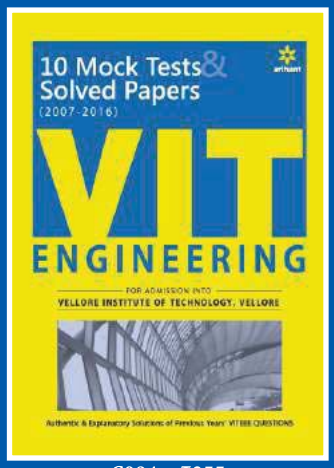
14 YEARS' सॉल्वड पेपर्स JEE Main
14 YEARS' SOLVED PAPERS JEE Main
CHAPTERWISE SOLUTIONS JEE Main PHYSICS
CHAPTERWISE SOLUTIONS JEE Main CHEMISTRY
CHAPTERWISE SOLUTIONS JEE Main MATHEMATICS

14 YEARS' SOLVED PAPERS for JEE Main

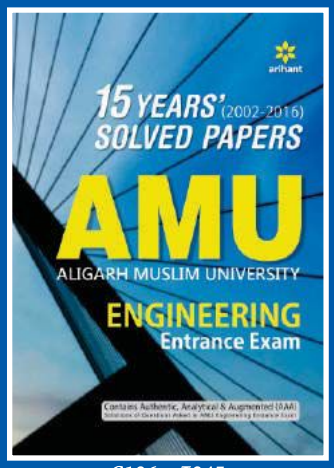
PRACTICE SETS For JEE MAIN & ADVANCED


TEST DRIVE FOR JEE MAIN 2017 15 PRACTICE SETS
TEST DRIVE FOR JEE MAIN 2017 15 प्रैक्टिस सेट्स
TEST DRIVE FOR JEE ADVANCED 2017 10 PRACTICE SETS

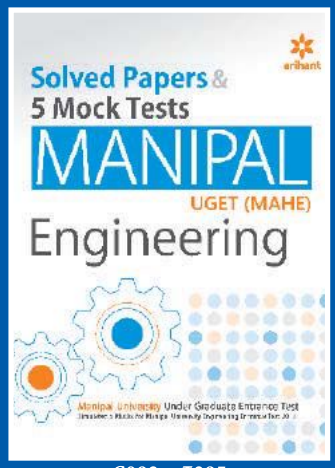
SOLVED PAPERS
for Engineering Entrances



C084 ₹355

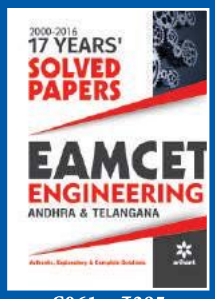


C136 ₹345



C092 ₹395

Andhra Pradesh



C061 ₹395

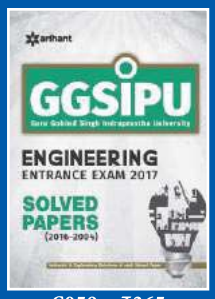
Bihar

C042 ₹320

Jharkhand

C045 ₹325

Delhi



C059 ₹365

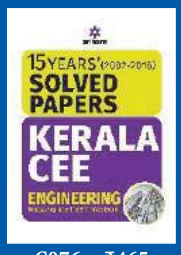
J & K

C091 ₹385

Karnataka

C032 ₹365

Kerala



C076 ₹465

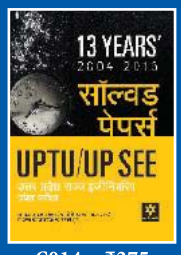
Maharashtra

C107 ₹310

UP

C072 ₹395

UP



C014 ₹375

West Bengal

C075 ₹415

www.arihantbooks.com

RNI-UP ENG/2008/18412 Postal Licence No. MEERUT-114/2011-2013